# TOPIC-1

## Simultaneous Equations

**Revision Notes:**

**1. Simultaneous Linear and Non–Linear Equations in Two Unknowns**

Use the substitution method. Express one unknown of the linear equation in terms of the other. Then we substitute this unknown in the non–linear equation to obtain a quadratic equation in one unknown which can be solved in the usual way.

# TOPIC- 2

## Surds, Indices and Logarithms



**Revision Notes:**

**1. Definition of surds**

When the root of a number or expression cannot be exactly determined, the root is called a SURD, e.g., $\sqrt{2}$ or $\sqrt{3}$ etc. A surd is also called an IRRATIONAL expression.

**2. Operations on Surds**

(a) $a\sqrt{x} + b\sqrt{x} = (a+b)\sqrt{x}$ e.g., $2\sqrt{3} + 4\sqrt{3} = (2+4)\sqrt{3} = 6\sqrt{3}$

(b) $a\sqrt{x} + b\sqrt{x} = (a-b)\sqrt{x}$ e.g., $2\sqrt{3} - 4\sqrt{3} = (2-4)\sqrt{3} = -2\sqrt{3}$

(c) $\sqrt{x} \times \sqrt{y} = \sqrt{x \times y}$ e.g., $\sqrt{3} - \sqrt{5} = \sqrt{3 \times 5} = \sqrt{15}$

(d) $\dfrac{\sqrt{x}}{\sqrt{y}} = \sqrt{\dfrac{x}{y}}$ e.g., $\dfrac{\sqrt{10}}{\sqrt{2}} = \sqrt{\dfrac{10}{2}} = \sqrt{5}$

**3. Rationalizing the Denominator**

It is usual not to write surds in the denominator of a fraction when this can be avoided. The process of clearing irrational numbers is called rationalization.

(a) $\dfrac{\sqrt{x}}{\sqrt{y}} = \dfrac{\sqrt{x}}{\sqrt{y}} \times \dfrac{\sqrt{y}}{\sqrt{y}} = \dfrac{\sqrt{xy}}{y}$

(b) $\frac{1}{\sqrt{x}+\sqrt{y}} = \frac{1}{\sqrt{x}+\sqrt{y}} \times \frac{\sqrt{x}-\sqrt{y}}{\sqrt{x}-\sqrt{y}} = \frac{\sqrt{x}-\sqrt{y}}{x-y}$

**Note:**

$\sqrt{x}+\sqrt{y}$ and $\sqrt{x}-\sqrt{y}$ are called conjugate surds. Their product, $\left(\sqrt{x}+\sqrt{y}\right)\left(\sqrt{x}-\sqrt{y}\right) = x - y$, is a rational number.

**4. Law of Indices**

If $a$ and $b$ are positive and $m$ and $n$, any rational indices, then

(a) $a^m \times a^n = a^{m+n}$

(b) $a^m \div a^n = a^{m-n}$

(c) $(a^m)^n = a^{mn}$

(d) $a^m \times b^m = (ab)^m$

(e) $a^m \div b^m = \left(\frac{a}{b}\right)^m$

(f) $a^0 = 1,\ a \neq 0$

(g) $a^{-n} = \frac{1}{a^n}$

(h) $a^{\frac{1}{n}} = \sqrt[n]{a}$

(i) $a^{\frac{m}{n}} = \sqrt[n]{a^m} = \left(\sqrt[n]{a}\right)^m$

**Note:**

When $a > 0$, $a^x > 0$ for all real values of $x$.

**5. Logarithms**

If $a^x = b,\ a > 0$ (Index form)

Then $\log_a b = x$ (Logarithmic form)

The logarithm to the base 10 is known as common logarithm.

The logarithm to the base e (= 2:718) is known as the natural logarithm.

**Note:** $\log_{10} x$ is also written as $\lg x$.

$\log_e x$ is also written as $\text{In}\, x$.

**6. Laws of Logarithms**

(a) $\log_a xy = \log_a x + \log_a y$

(b) $\log_a \frac{x}{y} = \log_a x - \log_a y$

(c) $\log_a x^n = n \log_a x$

(d) $\log_a 1 = 0$

(e) $\log_a a = 1$

(f) $\log_a x = \dfrac{\log_b x}{\log_b a}$, $\log_a x = \dfrac{1}{\log_x a}$ (change of base)

**Note:** $\log_a x$ exists if a > 0 and $x > 0$.

**Worked Examples:**

***Example 1***

Simplify the following.

(a) $\sqrt{50} + \sqrt{2} - 2\sqrt{18} + \sqrt{8}$ (b) $\left(2+\sqrt{3}\right)^2 - \left(3-\sqrt{3}\right)^2$

***Solution:***

(a) $\sqrt{50} + \sqrt{2} - 2\sqrt{18} + \sqrt{8} = \sqrt{25\times 2} + \sqrt{2} - 2\sqrt{9\times 2} + \sqrt{4\times 2}$

$= 5\sqrt{2} + \sqrt{2} - 6\sqrt{2} + 2\sqrt{2}$

$= 2\sqrt{2}$

(b) $\left(2+\sqrt{3}\right)^2 - \left(3-\sqrt{3}\right)^2 = 2^2 + 2(2)(\sqrt{3}) + (\sqrt{3})^2 - [3^2 - 2(3)(\sqrt{3}) + (\sqrt{3})^2]$

$= 4 + 4\sqrt{3} + 3 - [9 - 6\sqrt{3} + 3]$

$= 7 + 4\sqrt{3} - [12 - 6\sqrt{3}]$

$= 7 + 4\sqrt{3} - 12 + 6\sqrt{3}$

$= 10\sqrt{3} - 5$

# TOPIC- 3

# Quadratic Equations and Inequalities

**Revision Notes:**

### Nature of Roots of a Quadratic Equation

Consider the quadratic equation $ax^2 + bx + c = 0$ where $a$, $b$ and $c$ arc constants and $a \neq 0$.

The following formula can be used to solve the quadratic equation

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

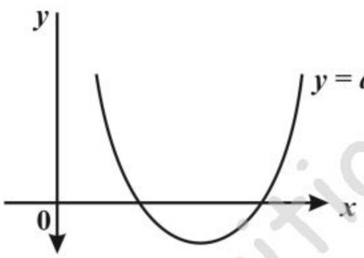
### Types of Roots

From the above formula, we find that, in general, there are two roots for every quadratic equation, but the two roots may be unequal or equal.

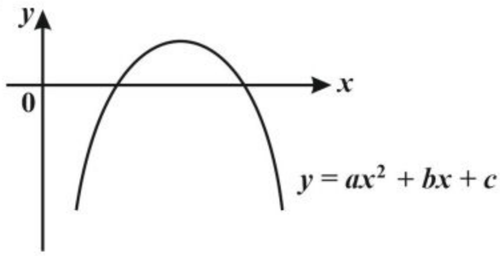
The roots of the equation will depend upon the value of $b^2 - 4ac$. The value $b^2 - 4ac$ is called the discriminant, of the quadratic equation $ax^2 + bx + c = 0$.

There are three types of roots of the quadratic equation

(a) If $b^2 - 4ac > 0$, the two roots are real and distinct (or 2 real unequal root)

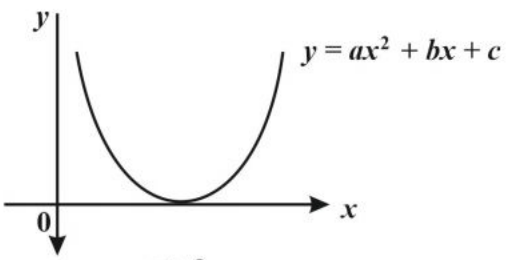



$a > 0$

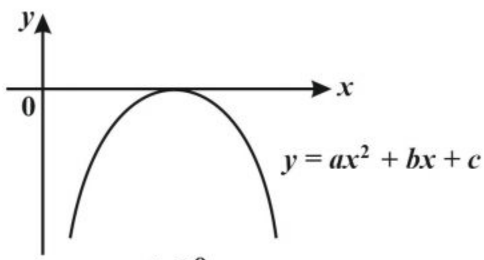



$a < 0$

(b) If $b^2 - 4ac = 0$, the two roots are real and equal (or coincident / repeated roots)

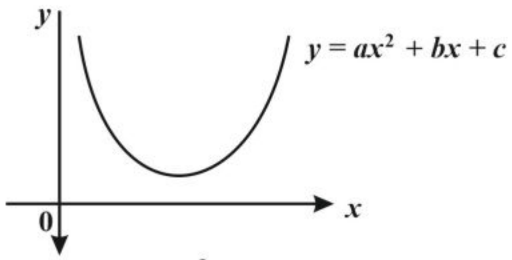



$a > 0$

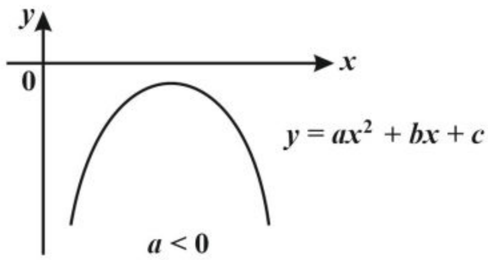



$a < 0$

(c) If $b^2 - 4ac < 0$, the two roots are complex or imaginary (i.e., no real roots)

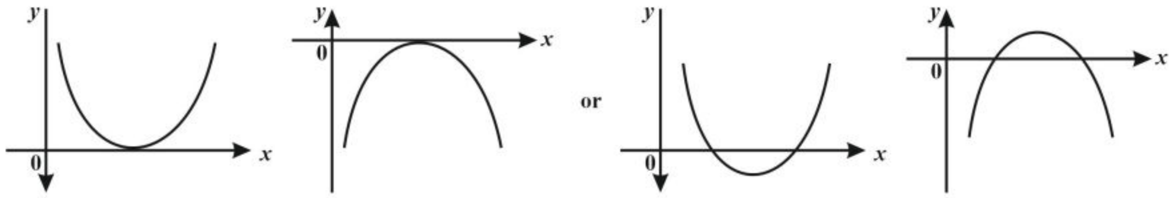



$a > 0$




$a < 0$

**Note:** If $b^2 - 4ac \geq 0$, the two roots are real (i.e., the roots are real and equal or real and unequal)




**3. Conditions for the Intersection of a Straight Line and a Curve**

If the equation of a straight line is $y = mx + c$ and the equation of a curve is $y = ax^2 + bx + d$, where $a$, $b$, $c$, $d$ and $m$ are constants.

At the point of intersection,

$$ax^2 + bx + d = mx + c$$

$$ax^2 + (b - m)x + (d - c) = 0 \qquad (1)$$

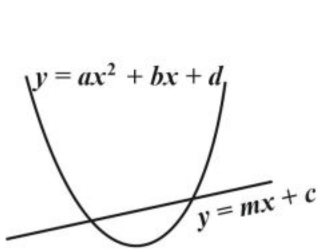
The solution from the equation (1) will give the $x$–coordinate of the points of intersection of these two graphs.

In equation (1), if

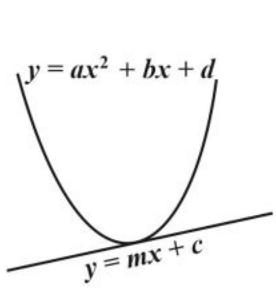
(i) the **discriminant > 0**, then the line intersects the curve at two distinct points,

(ii) the **discriminant = 0**, then the line will only meet the curve at one point and the line is a tangent to the curve,

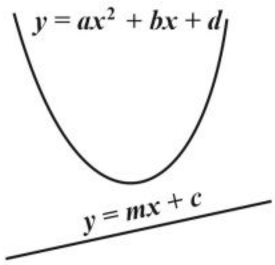
(iii) the **discriminant < 0**, then the line does not meet the curve.




**(Discriminant > 0)**




**(Discriminant = 0)**

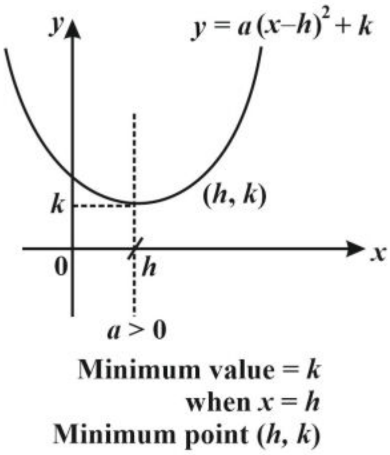


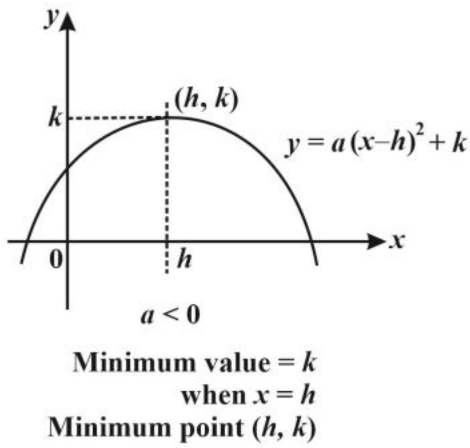

**(Discriminant < 0)**

**4. Maximum / Minimum Value of a Quadratic Function**

We can express a given quadratic function $f(x) = ax^2 + bx + c$ in the form of $f(x) = a(x - h)^2 + k$ using the completing square method.

A quadratic function expressed in the form of $a(x - h)^2 + k$ has a maximum or minimum value (depending on the value of $a$) of $k$ when $x = h$.



# TOPIC- 4

## Partial Fractions

**Revision Notes:**

For two polynomials in $x$, $N(x)$ and $D(x)$, the rational function $\frac{N(x)}{D(x)}$ is a proper fraction if the degree of $N(x)$ is strictly less than that of $D(x)$. Otherwise, it is an improper fraction.

e.g., $\frac{1}{x^2-4}, \frac{2x^2+3}{x(x-1)^2}$ and $\frac{1}{x(x^2+x+1)}$ are proper fractions

$\frac{x^5}{x^2-1}$ is an improper fraction

An improper rational function can be converted, by division or otherwise, into a sum of polynomial and a proper fraction.

e.g., $\frac{x^5}{x^2-1} = x^3 + x + \frac{x}{x^2-1}$

$$\begin{array}{r l}
 & x^3 + x \\
x^2 - 1 \,\big) & x^5 \\
 & \underline{x^5 - x^3} \\
 & x^3 \\
 & \underline{x^3 - x} \\
 & x
\end{array}$$

A single rational function can be expressed as a sum of two or more simpler fractions, called partial fractions.

To express $\frac{N(x)}{D(x)}$ in partial fractions:

Step 1: Check whether $\frac{N(x)}{D(x)}$ is proper. If not, express it as a sum of polynomial and a proper fraction using long division or manipulate the numerator so that simple cancellation can be done.

Step 2: Check if the denominator is completely factorised.

Step 3: Express in partial fractions using cases 1, 2 and 3.

| Case | Denominator of fraction with | Algebraic fraction | Expression used |
|---|---|---|---|
| 1 | Linear factor | $\frac{mx+n}{(ax+b)(cx+d)}$ | $= \frac{A}{ax+b} + \frac{C}{cx+d}$ |
| 2 | Repeated factors | $\frac{mx+n}{(ax+b)(cx+d)^2}$ | $= \frac{A}{ax+b} + \frac{B}{cx+d} + \frac{C}{(cx+d)^2}$ |
| 3 | Quadratic factor which does not factorise | $\frac{mx+n}{(ax+b)(x^2+c^2)}$ | $= \frac{A}{ax+b} + \frac{Bx+C}{x^2+c^2}$ |

Step 4: Solve for unknown constants by

(i) substituting values of $x$ and/or

(ii) comparing coefficients of like terms.

# TOPIC- 6

# Binomial Theorem

**Revision Notes:**

**1 The Binomial Theorem**

If $n$ is a positive integer,

$$(x+y)^n = x^n + nx^{n-1}y + \frac{n(n-1)}{1\times 2}x^{n-2}y^2 + \frac{n(n-1)(n-2)}{1\times 2\times 3}x^{n-3}y^3 + \ldots + y^n$$

**2 Binomial Theorem expressed in terms of Combination Notation**

The combination notation ${}^nC_r$ is denoted by ${}^nC_r = \frac{n(n-1)(n-2)\ldots(n-r+1)}{1\times 2\times 3\times\ldots\times r}$

Using the combination notation, the expression $(x+y)^n$ can be written as

$$(x+y)^n = x^n + {}^nC_1x^{n-1}y + {}^nC_2x^{n-2}y^2 + {}^nC_3x^{n-3}y^3 + \ldots + {}^nC_rx^{n-r}y^r + \ldots + y^n$$

3 The (r + 1) term = ${}^nC_rx^{n-r}y^r$

4 The term independent of $x$ is the constant term

**Worked Examples:**

***Example 1***

The expansion of $(1+px)^n$, where $n > 0$, by the binomial theorem is $1 + 20x + 45p^2x^2 + kx^3 + \ldots$ Calculate $n$, $p$ and $k$.

***Solution:***

$$(1+px)^n = 1 + npx + \frac{n(n-1)}{2}p^2x^2 + \frac{n(n-1)(n-2)}{6}p^3x^3 + \ldots$$

$$= 1 + 20x + 45p^2x^2 + kx^3 + \ldots\ldots\ldots$$

Equating the coefficients of $x$, $x^2$ and $x^3$,

$np = 20$ (1)

$\frac{n(n-1)}{2} = 45$ (2)

$k = \frac{n(n-1)(n-2)}{6}p^3$ (3)

From (2): $n(n-1) = 90$

$n^2 - n - 90 = 0$

$(n-10)(n+9) = 0$

$n = 10$ or $n = -9$ (rejected)

From (1): $10p = 20$

$p = 2$

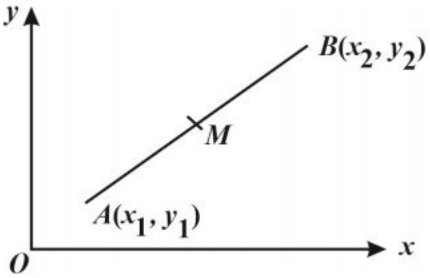
From (3): $k = \dfrac{10(9)(8)}{6} \times 2^3 = 960$

# TOPIC-7

# Coordinate Geometry

**Revision Notes:**

**1. Distance and mid–point**

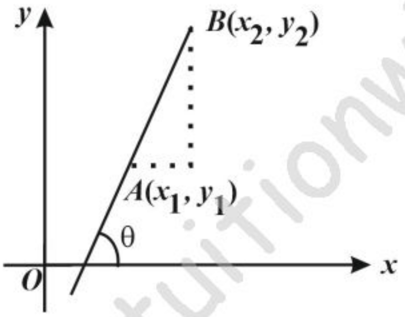



The distance $AB$ between two points A $(x_1, y_1)$ and $B$ $(x_2, y_2)$ is $\sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$ .

The mid–point $M$ of $A$ $(x_1\ y_1)$ and $B$ $(x_2, y_2)$ is $\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$

**2. Gradient of a straight line**

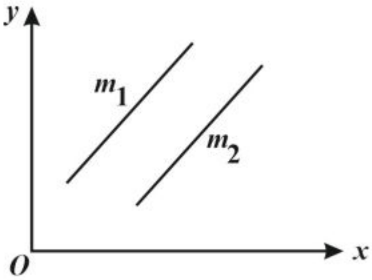
**Definition**




Gradient of a line is defined as the tangent of the angle which the line makes with the positive direction of the $x$–axis.

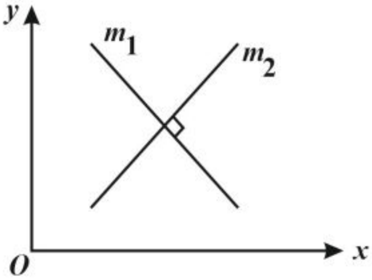
Given $A$ $(x_1, y_1)$ and $B$ $(x_2, y_2)$ then the gradient of $AB$ is $\frac{y_2 - y_1}{x_2 - x_1} = \tan\theta$.

**Gradient of parallel lines**




The gradient of a parallel lines are equal, i.e., $m_1 = m_2$.

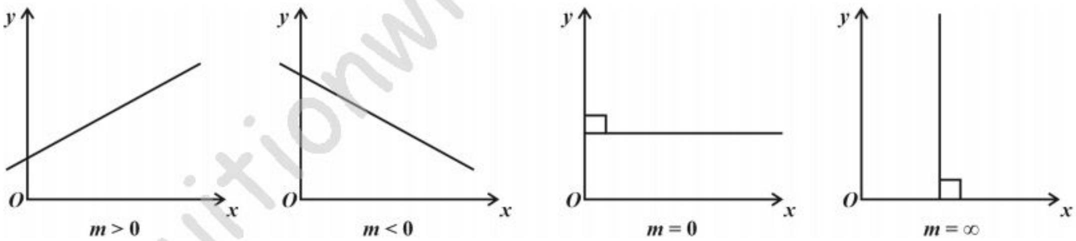
**Gradients of perpendicular lines**



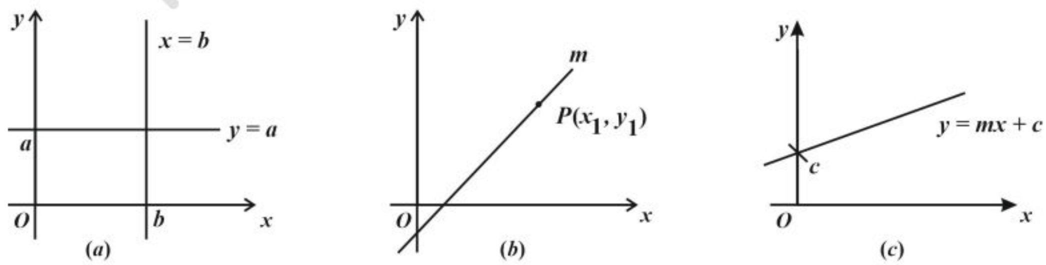

The product of the gradients of two perpendicular lines is – 1.

i.e., $m_1 \times m_2 = -1$ or $m_1 = \frac{1}{m^2}$

**Gradients of different types of straight lines**




**3. The straight lines**




**(a) Lines parallel to the coordinates axes**

Equation of straight line parallel to the $x$–axis is $y = a$ where a is $a$ constant

Equation of straight line parallel to the $y$–axis is $x = b$ where b is $a$ constant

**(b) Straight line of gradient and point form**

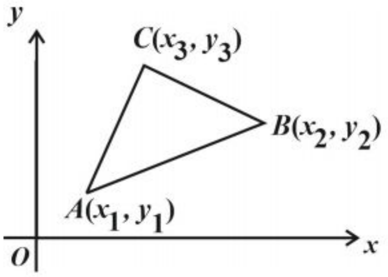
If a straight line passing through a point $P(x_1, y_1)$ and having a gradient $m$, then its equation is $y - y_1 = m(x - x_1)$.

**(c) Gradient form of straight line**

If a straight line makes a $y$–intercept of length $c$ and having a gradient $m$, then its equation is $y = mx + c$.

**4. Area of Plane Figures (Given the vertices)**

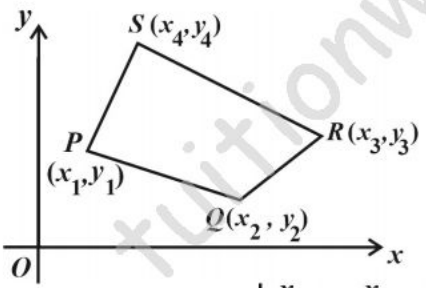
(a)




$$\text{Area of } \Delta ABC = \frac{1}{2}\begin{vmatrix} x_1 & x_2 & x_3 & x_1 \\ y_1 & y_2 & y_3 & y_1 \end{vmatrix}$$

$$= \frac{1}{2} \left| [(x_1y_2 + x_2y_3 + x_3y_1) - (y_1 x_2 + y_2x_3 + y_3 x_1)] \right|$$

(b)




$$\text{Area of } \Delta PQRS = \frac{1}{2}\begin{vmatrix} x_1 & x_2 & x_3 & x_4 & x_1 \\ y_1 & y_2 & y_3 & y_4 & y_1 \end{vmatrix}$$

$$= \frac{1}{2} \left| [(x_1y_2 + x_2y_3 + x_3y_4 + x_4y_1) - (y_1x_2 + y_2x_3 + y_3x_4 + y_4x_1)] \right|$$

**Note:** When the vertices are listed in anticlockwise direction, the area is positive. If the vertices are listed in clockwise direction, the area is negative. Discard the negative sign.

# TOPIC- 8

## Linear Law

**Revision Notes:**

Non–linear functions in variables $x$ and $y$ can be reduced to linear functions in the form of

$Y = mX + c$

where $m$ = gradient

$c$ = Y–intercept

$X$ and $Y$ are expressions in $x$ and/or $y$.

The table shows how some non–linear functions can be reduced to the linear form :

| | Functions | $Y$ | $X$ | $m$ | $c$ |
|---|---|---|---|---|---|
| | $(a)\ y = ax^2 + b$ | y | $x^2$ | $a$ | $b$ |
| | $(b)\ y = \frac{a}{x} + b$ | y | $\frac{1}{x}$ | $a$ | $b$ |
| | $(c)\ \frac{1}{y} = ax^2 + b$ | $\frac{1}{y}$ | $x^2$ | $a$ | $b$ |
| | $(d)\ y = a\sqrt{x} + \frac{b}{\sqrt{x}}$ change to<br>$\sqrt{x}y = ax + b$ | $y\sqrt{x}$ | $x$ | $a$ | $b$ |
| | $(e)\ y = ax^2 + bx$ change to<br>$\frac{y}{x} = ax + b$ | $\frac{y}{x}$ | $x$ | $a$ | $b$ |
| | $(f)\ y = ab^x$ change to<br>$\lg y = \lg(ab^x)$<br>$\lg y = \lg a + x \lg b$<br>$\lg y = (\lg b)\,x + \lg a$ | $\lg y$ | $x$ | $\lg b$ | $\lg a$ |
| | $(g)\ y = ax^b$ change to<br>$\lg y = \lg(ax^b)$<br>$\lg y = \lg a + b \lg x$<br>$\lg y = b \lg x + \lg a$ | $\lg y$ | $\lg x$ | $b$ | $\lg a$ |

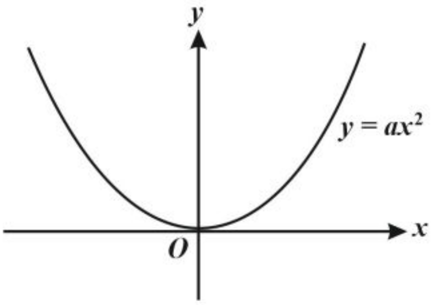
# TOPIC - 9

# Further Coordinate Geometry

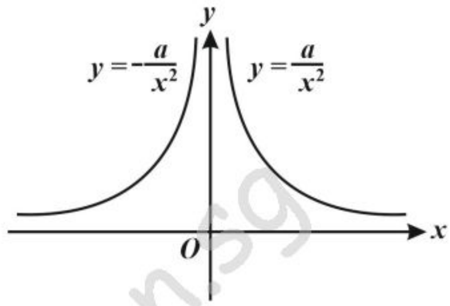
**Revision Notes:**

1. Graphs of the form $y = ax^n$, $a > 0$

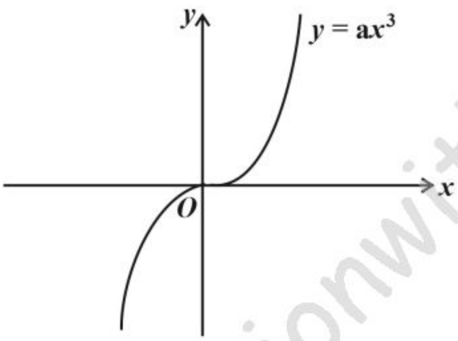
(a) $n = 2$



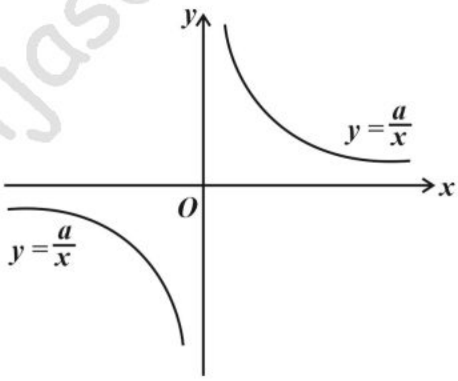

(b) $n = -2$



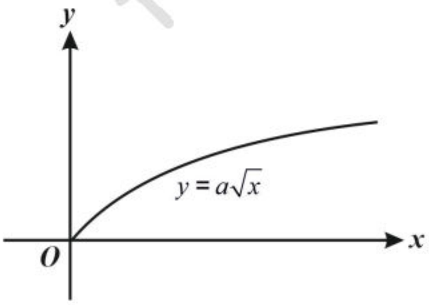

(c) $n = 3$



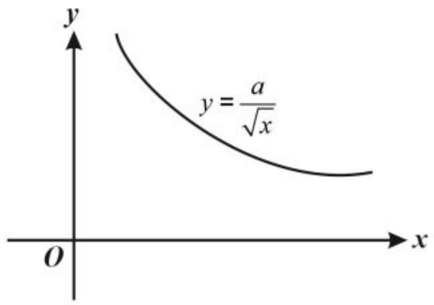

(d) $n = -1$



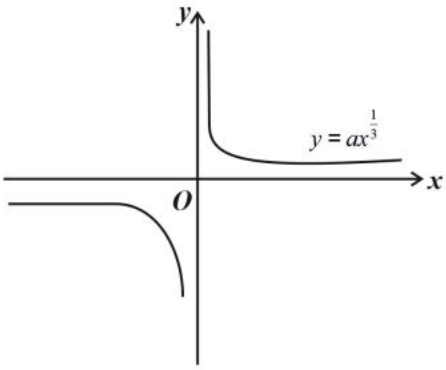

(e) $n = \frac{1}{2}$



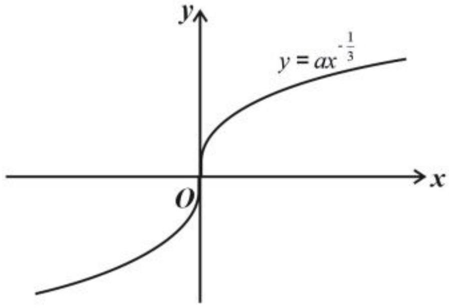

(f) $n = -\frac{1}{2}$



(g) $n = \frac{1}{3}$

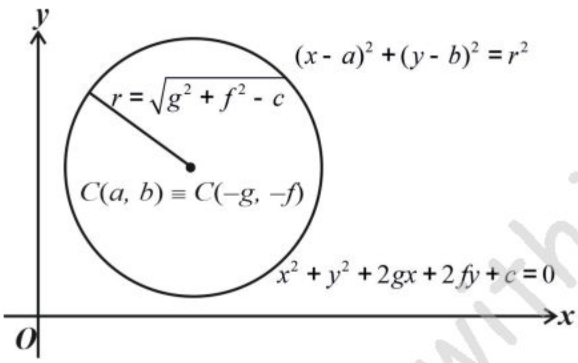



(h) $n = -\frac{1}{3}$




**Note:** The graphs are symmetrical about the $x$–axis.

**Circles**




Centre: $C\ (a,\ b) = C\ (g, f)$

Radius: $\text{r} = \sqrt{g^2 + f^2 - c}$

Equation: $(x - \text{h})^2 + (\text{y} - \text{k})^2 = \text{r}^2$

$x^2 + y^2 + 2gx + 2fy + \text{c} = 0$

**Worked Examples:**

***Example 1***

Find the centre and radius of the circle $x^2 + y^2 + 2x - 4y - 20 = 0$.

Find the equation of the tangent at $P$ (2, 6).

***Solution:***

$x^2 + y^2 + 2\text{x} - 4\text{y} - 20 = 0$

$(x_T + 1)^2 - 1 + (y - 2)^2 - 4 - 20 = 0$

$(\text{x}+1)^2 + (\text{y} - 2)^2 = 25$

Centre is (–1, 2) and radius is 5 units.

# TOPIC- 10

## Geometrical Proofs

**Revision Notes:**

**1. Congruent Triangles**

Tests of congruency:

(a) SSS

(b) SAS

(c) AAS or ASA

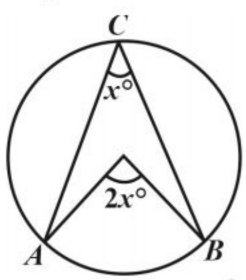
(d) RHS

**2. Similar Triangles**

(a) All corresponding ∠s are equal.

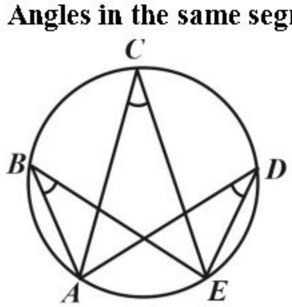
(b) All corresponding sides are proportional

**3. Angle at Centre**



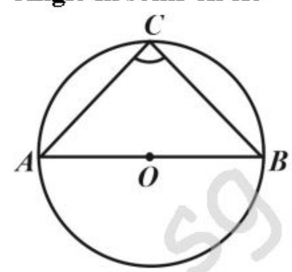

$\angle AOB = 2\angle ACB$

**4. Angles in the same segment**



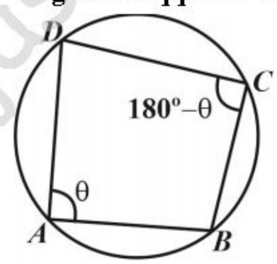

$ABE = \angle ACE = \angle ADE$

**5. Angle in semi-circle**




$\angle ACB = 90°$

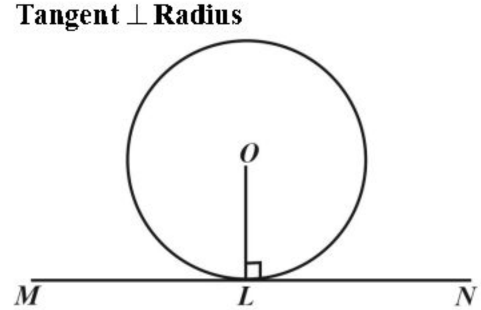
**6. Angles in Opposite Segments**




$\angle A + \angle C = 180°$

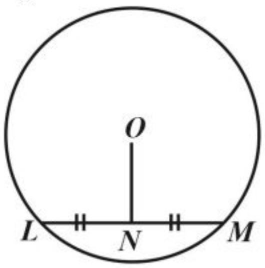
$\angle B + \angle D = 180°$

**7. Tangent ⊥ Radius**




$OL \perp MN$

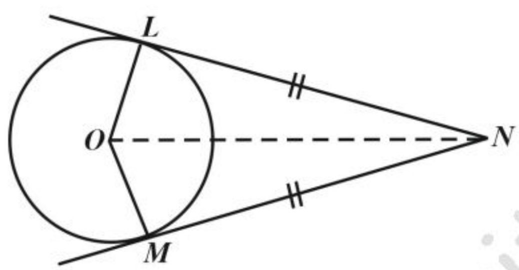
**8. Perpendicular bisector of chord passes through centre**




$ON \perp LM$

$LN = MN$

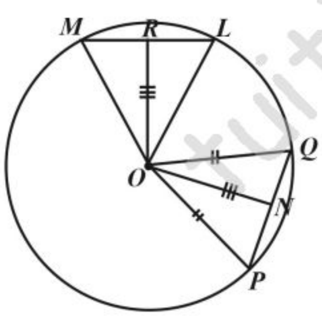
**9. Tangents from external points**




$LN = NM$

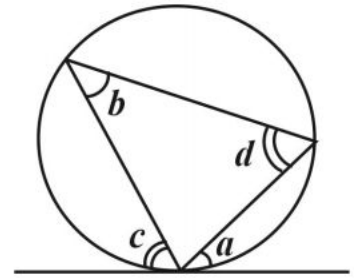
$\angle LNO = \angle MNO$

**10. Equal chords are equidistant from centre**




$PQ = LM \Leftrightarrow ON = OR$

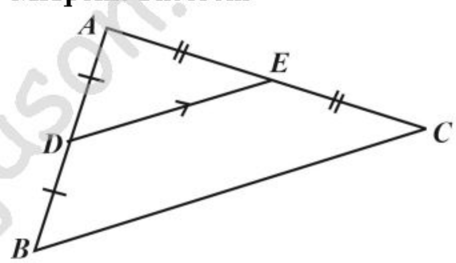
**11. Alternate Segment Theorem**




$\angle a = \angle b$

$\angle c = \angle d$

**12. Midpoint Theorem**




If $D$ and $E$ are midpoints of $AB$ and $AC$, then $DE \parallel BC$ and $DE = \frac{1}{2}BC$.

# TOPIC- 11

# Remainder and Factor Theorem

**Revision Notes**

**1. Remainder Theorem**

If a polynomial f(x) is divided by $(x - a)$, the remainder is $f(a)$

**2. Factor Theorem**

If a polynomial $f(x)$ is divided by $(x - a)$ and the remainder $f(a) = 0$, then $(x - a)$ is a factor of $f(x)$. Conversely, if $(x - a)$ is a factor of $f(x)$, then $f(a) = 0$.

**3. Solving Cubic Equations**

To find the solution of a given cubic equation, $ax^3 + bx^2 + cx + d = 0$, where $a$, $b$, $c$ and $d$ are constant

Step 1: Find one factor by trial and error using the Factor Theorem.

Step 2: Use either **Long Division** or the **Inspection** method to find the other factors.

Step 3: Solve the equations for the roots.

**4. Identity**

A mathematical identity is a relation which is true for all values of the unknown.

e.g. $(x + 1)^2 = x^2 + 2x + 1$ for all values of $x$.

**Worked Examples:**

***Example 1***

The expression $2x^3 + ax^2 - 3x + b$ is exactly divisible by $x^2 + x - 2$. Find the value of a and of $b$.

***Solution:***

Let $f(x) = 2x^3 + ax^2 - 3x + b$

$x^2 + x - 2 = (x + 2)(x - 1)$

$f(x)$ is exactly divisible by $x^2 + x - 2 \Rightarrow (x + 2)$ and $(x - 1)$ are factors of $f(x)$.

$f(-2) = 0$

$f(-2) = 0$

$2(-2)^3 + a(-2)^2 - 3(-2) + b = 0$

$-16 + 4a + 6 + b = 0$

$4a + b = 10 \qquad (1)$

$f(1) = 0$

$2(1)^3 + a(1)^2 - 3(1) + b = 0$

$2 + a - 3 + b = 0$

$a + b = 1 \quad (2)$

$(1) - (2) \quad 3a = 9$

$a = 3$

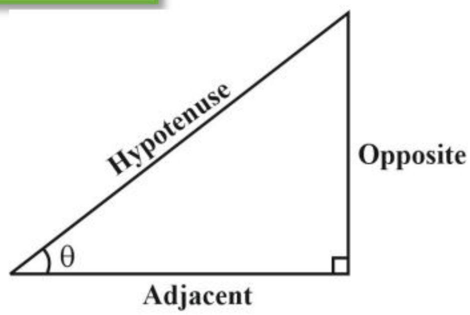
Sub, $a = 3$ into (2), $b = 1 - 3 = 2$

$\therefore\ a = 3, b = -2$

# TOPIC- 12

# Trigonometry

**Revision Notes:**



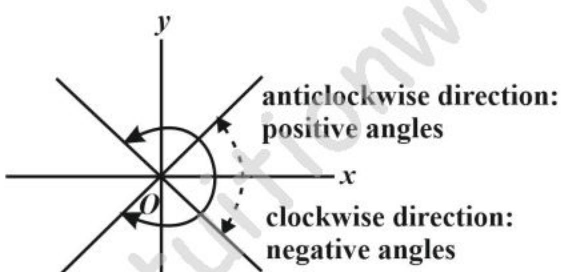

$\sin\theta = \dfrac{\text{Opposite}}{\text{Hypotenuse}}$ $\quad\cos\theta = \dfrac{\text{Adjacent}}{\text{Hypotenuse}}$ $\quad\tan\theta = \dfrac{\text{Opposite}}{\text{Adjacent}}$

**1. Relation between Radian and Degree**

$\pi$ radians = 180°

$\Rightarrow 1 \text{ radian} = \dfrac{180°}{\pi}, \; 1° = \dfrac{\pi}{180°} \text{ radian}$

**2. Positive and Negative Angles**




Angie measured from $ox$ in anticlockwise sense is positive angle.

Angle measured from $ox$ in clockwise sense is negative angle.

**3. Signs of Trigonometric Ratios in the Four Quadrants**

| | |
|---|---|
| 2nd quadrant<br>Sine Positive | 1st quadrant<br>All Positive |
| Tangent Positive<br>3rd quadrant | Cosine Positive<br>4th quadrant |

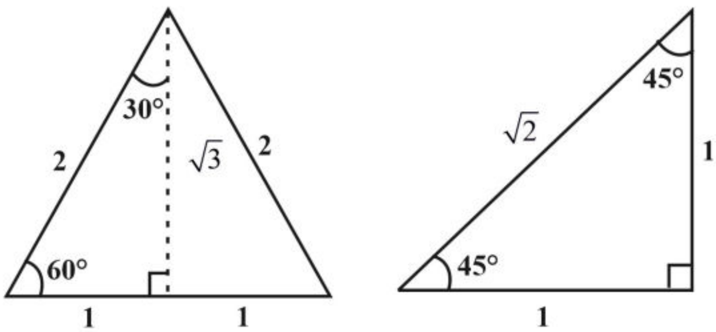
**Note:** To remember the positive ratio use

| | |
|---|---|
| **S** | **A** |
| **T** | **C** |

**4. Trigonometrical Ratio of Special Angles**

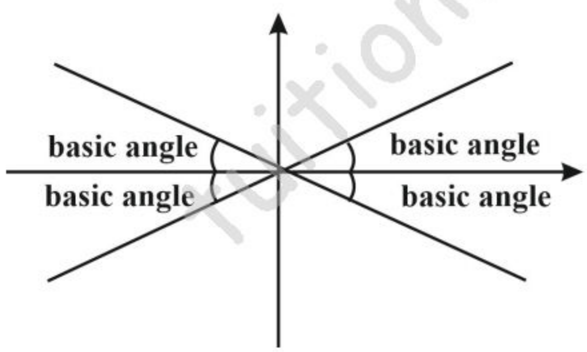
For 30° or 60°, consider an equilateral triangle of side 2 units.

For 45°, consider a right-angled isosceles triangle.




| $\theta$ | 30° | 45° | 60° |
|---|---|---|---|
| $\sin\theta$ | $\frac{1}{2}$ | $\frac{1}{\sqrt{2}}$ | $\frac{\sqrt{3}}{2}$ |
| $\cos\theta$ | $\frac{\sqrt{3}}{2}$ | $\frac{1}{\sqrt{2}}$ | $\frac{1}{2}$ |
| $\tan\theta$ | $\frac{1}{\sqrt{3}}$ | 1 | $\sqrt{3}$ |

**6. The basic angle is the acute angle between a rotating radius about the origin and the $x$–axis**




**7. Trigonometrical Identities**

(a) $\tan\theta = \frac{\sin\theta}{\cos\theta}$

$\cot\theta = \frac{\cos\theta}{\sin\theta} = \frac{1}{\tan\theta}$

(b) $\sin(-\theta) = -\sin\theta$

$\cos(-\theta) = \cos\theta$

(c) $\sin(90° - \theta) = \cos\theta$

$\cos(90° - \theta) = \sin\theta$

$\sec\theta = \dfrac{1}{\cos\theta}$ $\tan(-\theta) = -\tan\theta$ $\tan(90° - \theta) = \cot\theta = \dfrac{1}{\tan\theta}$

$\text{cosec}\,\theta = \dfrac{1}{\sin\theta}$

(d) $\sin(180° - \theta) = \sin\theta$

$\cos(180° - \theta) = -\cos\theta$

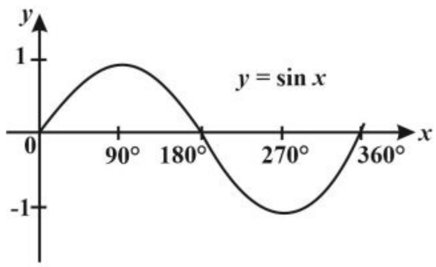
$\tan(180° - \theta) = -\tan\theta$

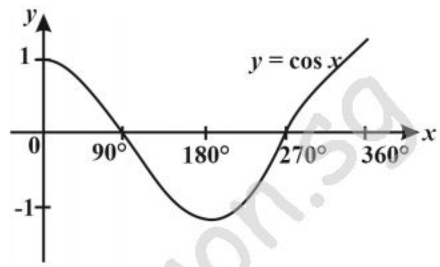
(e) $\sin^2\theta + \cos^2\theta = 1$

$1 + \tan^2\theta = \sec^2\theta$

$1 + \cot^2\theta = \text{cosece}^2\theta$

**8. Graphs of sin $x$, cos $x$ and tan $x$**



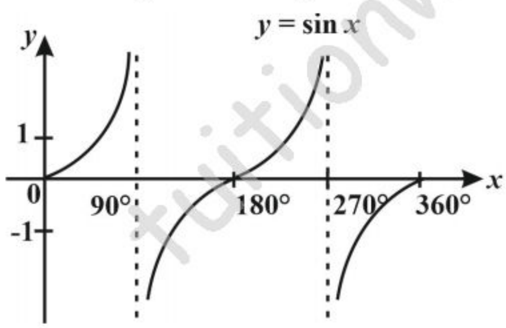

From the above graphs, we note the following:

(a) The graphs of sin $x$ and cos $x$ will repeat itself after an interval 360° or $2\pi$;

i.e., the period = 360° or $2\pi$.

(b) The maximum value and minimum values of sin $x$ and cos $x$ are 1 and $-1$ respectively.

i.e., the magnitude or amplitude = 1.



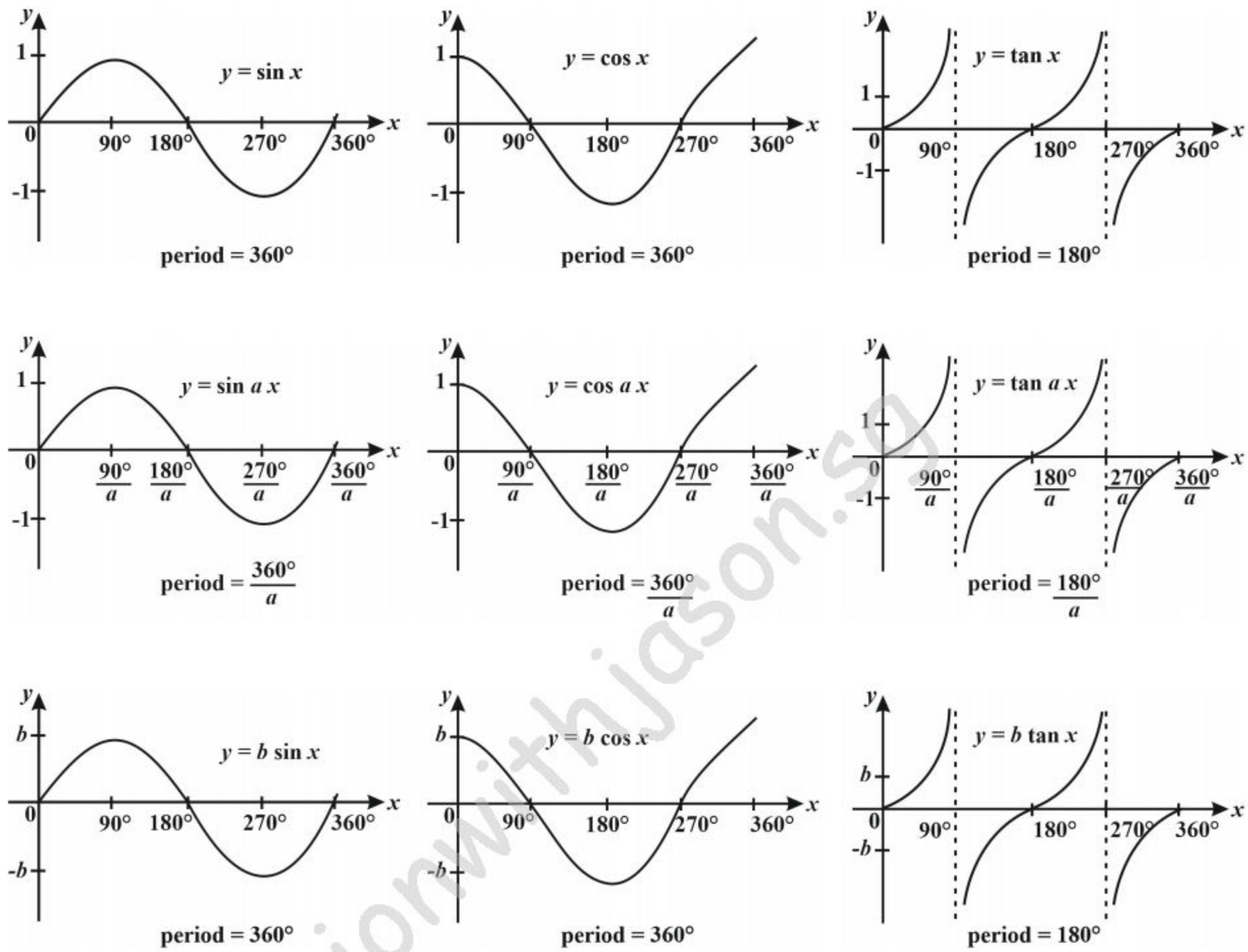

From the graph of $y = \tan x$, we note the following:

(a) tan $x$ is periodic with period 180° or $n$.

(b) tan $x$ has no maximum or minimum value. It can take all real values.

(c) tan $x$ is undefined when 0 = 90° $\left(=\dfrac{\pi}{2}\right)$ or 270° $\left(=\dfrac{3\pi}{2}\right)$. The approach the vertical line drawn through $x = 90°$, 270°. Such that vertical lines through $x = 90°$ and 270° are called the asymptotes of the graph.

## 9. More Trigonometrical Graphs




$= a \sin bx + c$ period $= \dfrac{360^\circ}{b}\left(or \dfrac{2\pi}{b}\right)$ amplitude $= a$

$= a \cos bx + c$ period $= \dfrac{360^\circ}{b}\left(or \dfrac{2\pi}{b}\right)$ amplitude $= a$

$= a \tan bx + c$ period $= \dfrac{180^\circ}{b}\left(or \dfrac{\pi}{b}\right)$

## 10. Principal Values

$-90^\circ \le \sin^{-1} y \le 90^\circ \left(or -\dfrac{\pi}{b} \le \sin^{-1} y \le \dfrac{\pi}{2}\right)$

$0^\circ \le \cos^{-1} y \le 180^\circ \left(or\, 0 \le \cos^{-1} y \le \pi\right)$

$-90^\circ < \tan^{-1} y < 90^\circ \left(or -\dfrac{\pi}{2} < \tan^{-1} y < \dfrac{\pi}{2}\right)$

# TOPIC - 13

# Further Trigonometry

**Revision Notes**

**1. Addition Formulae**

$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$

$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$

$\tan(A \pm B) = \dfrac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$

**2. Double Angle Formulae**

$\sin 2A = 2 \sin A \cos A$

$\cos 2A = \cos^2 A - \sin^2 A$

$= 2\cos^2 A - 1$

$= 1 - 2\sin^2 A$

$\tan 2A = \dfrac{2\tan A}{1 - \tan^2 A}$

**3. R–Formulae**

$a \sin x \pm b \cos x = R \sin(x \pm a)$

$a \cos x \pm b \sin x = R \cos(x \mp a)$

where $R = \sqrt{a^2 + b^2}$ and $\tan a = \dfrac{b}{a}$

# TOPIC - 14

# Differentiation

## Revision Notes

1. if $y = f(x)$ represents a line or curve, $\frac{dy}{dx}$ represents the gradient of the line or the curve at a point.

   The gradients of the curve at different points have different gradients.

2. $\frac{dy}{dx}$ or $f'(x)$ is also called the **first derivative, the derived function** or the **differential coefficient** of $y$ with respect to $x$.

3. $\frac{d^2y}{dx^2}$ is called the *second derivative*,

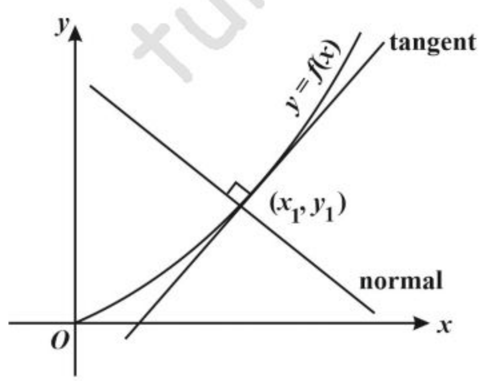
4. If $y = k$, where $k$ is a constant, $\frac{dy}{dx} = 0$

5. If $y = kx^n$, $\frac{dy}{dx} = knx^{n-1}$

6. If $y = u \pm v$ where $u = f(x)$, $v = g(x)$, $\frac{dy}{dx} = \frac{du}{dx} + \frac{dv}{dx}$

7. If $y = f(u)$, $u = g(x)$, $\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx}$ (Chain Rule)

   $\frac{dy}{dx} = \frac{1}{\frac{dy}{dx}}$




Equation of the tangent: $y - y_1 = m(x - x_1)$

Equation of the normal: $y - y_1 = -\frac{1}{m}(x - x_1)$ where $m = \frac{dy}{dx}$ at the point $(x_1, y_1)$

**8. Differentiation of a product (Product rule)**

If $y = uv$ where $u = \mathrm{f}(x)$ and $v = \mathrm{g}(x)$, then $\frac{dy}{dx} = u\frac{dv}{dx} + v\frac{du}{dx}$

**9. Differentiation of a quotient (Quotient rule)**

If $y = \frac{u}{v}$ where $u = \mathrm{f}(x)$ and $\mathrm{v} = g(x)$, then $\frac{dy}{dx} = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$

# TOPIC- 15

# Rate of Change

**Revision Notes**

$\frac{dy}{dx}$ is the rate of change of y with respect to $x$.

If $x$ and $y$ are given in terms of the time, the rates of change $\frac{dx}{dt}$ and $\frac{dy}{dt}$ are related by:

$$\frac{dy}{dx} = \frac{dy}{dt} \times \frac{dt}{dx}$$

**Worked Examples:**

***Example 1***

A viscous liquid is poured on to a flat surface. It forms a circular patch which grows at a steady rate of 5 $cm^2/s$. Find, in terms of $\pi$,

(a) the radius of the patch 20 seconds after pouring has commenced.

(b) the rate of increase of the radius at this instant

***Solution:***

(a) $A = \pi r^2$ where $r$ = radius of circle

$A$ = area of circle

After 20 s, $A = 5 \times 20 = 100$

$$\pi r^2 = 100$$

$$r^2 = \frac{100}{\pi}$$

$$r = \frac{10}{\sqrt{\pi}}$$

$\therefore$ The radius of the patch is $\frac{10}{\sqrt{\pi}}$ cm.

(b) $\frac{dA}{dr} = 2\pi r$

When $r = \frac{10}{\sqrt{\pi}}$, $\quad \frac{dA}{dr} = 2\pi\left(\frac{10}{\sqrt{\pi}}\right) = 20\sqrt{\pi}$

$$\frac{dr}{dt} = \frac{dr}{dA} \times \frac{dA}{dr}$$

$$= \frac{1}{20\sqrt{\pi}} \times 5$$

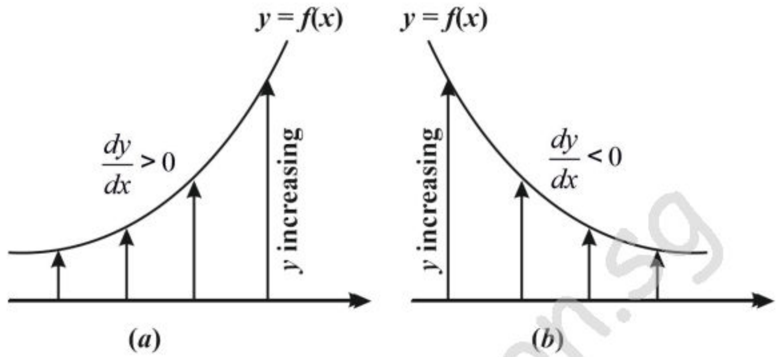
$$= \frac{1}{4\sqrt{\pi}}$$

$\therefore$ The rate of increase of the radius is $\frac{1}{4\sqrt{\pi}}$ cm/s.

# TOPIC-16

## Stationary Points, Maxima and Minima Problems

**Revision Notes:**

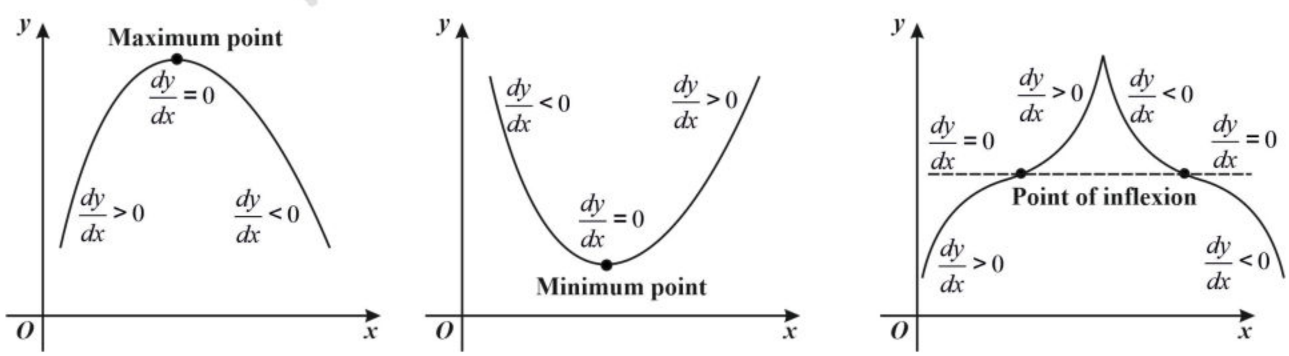



1. In Fig, (a), on any stretch of the curve $y = f(x)$ where $\frac{dy}{dx} > 0$, the curve slopes upwards. Hence $y$ increases as $x$ increases.

   Similarly, if $\frac{dy}{dx} > 0$ (Fig. (*b*)), $y$ decrease as $x$ increases.

2. A stationary point on a curve is defined as a point where the $\frac{dy}{dx} = 0$.

3. There are two types of stationary points:
   (a) turning points (maximum / minimum points)
   (b) points of inflexion



**4. Test for a maximum point, minimum point, point of inflexion**

| | 1st derivative test | 2nd derivative test |
|---|---|---|
| **Maximum point** | changes from positive to negative | $\frac{d^2y}{dx^2} < 0$ |
| **Minimum point** | changes from negative to positive | $\frac{d^2y}{dx^2} > 0$ |
| **Point of inflexion** | does not change sign | — |

**Note:** Maximum and minimum applies around the stationary point. The values of the function at this point are not necessarily the greatest and least values of the whole function.

# TOPIC -17

## Derivatives of Trigonometric Functions

**Revision Notes:**

1. $\frac{d}{dx}(\sin x) = \cos x$

$\frac{d}{dx}(\cos x) = -\sin x$

$\frac{d}{dx}(\tan x) = \sec^2 x$

2. $\frac{d}{dx}[\sin(ax + b)] = a\cos(ax + b)$

$\frac{d}{dx}[\cos(ax + b)] = -a\sin(ax + b)$

$\frac{d}{dx}[\tan(ax + b)] = a\sec^2(ax + b)$

where a and b are constants.

3. $\frac{d}{dx}(\sin^n x) = n\sin^{n-1} x\cos x$

$\frac{d}{dx}(\cos^n x) = -n\cos^{n-1} x\sin x$

$\frac{d}{dx}(\tan^n x) = n\tan^{n-1} x\sec^2 x$

**Worked Examples:**

***Example I***

Differentiate the following with respect to $x$.

(a) $\sin^3 2x$

(b) $\sqrt{1+3\cos^2 x}$

***Solution:***

(a) $\frac{d}{dx}(\sin^3 2x) = 3(\sin^2 2x)(\cos 2x)(2)$

$= 6 \sin^2 2x \cos 2x$

(b) $\frac{d}{dx}\sqrt{1+3\cos^2 x} = \frac{d}{dx}(1+3\cos^2 x)^{\frac{1}{2}}$

$= \frac{1}{2}(1+3\cos^2 x)^{-\frac{1}{2}}(6\cos x)(-\sin x)$

$= \frac{-3\sin x\cos x}{\sqrt{1+3\cos^2 x}}$

# TOPIC- 18

## Exponential and Logarithmic Functions

**Revision Notes:**

**1. Differentiation of Exponential Functions**

$$\frac{d}{dx} = \mathrm{e}^x$$

$$\frac{d}{dx}\,(\mathrm{e}^{ax+b}) = a e^{ax+b}$$ where $a$ and $b$ are constants

In general,

$$\frac{d}{dx}\,(\mathrm{e}^{\mathrm{f}(x)}) = \mathrm{e}^{\mathrm{f}(x)} \times \frac{d}{dx}\,[\mathrm{f}(x)]$$

**2. Differentiation of Logarithmic Functions**

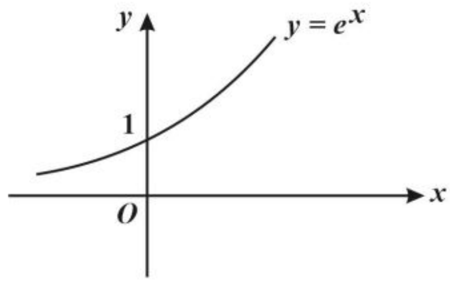
$$\frac{d}{dx}\,(\mathrm{In}\,x) = \frac{1}{x}$$

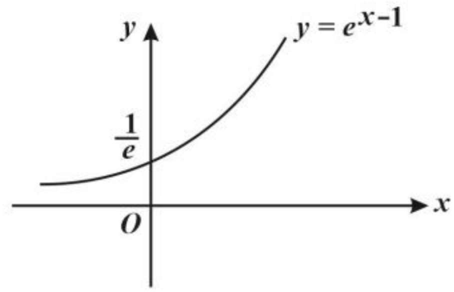
$$\frac{d}{dx}\,[\mathrm{In}\,(ax + b)] = \frac{a}{ax+b}$$ where a and b are constants

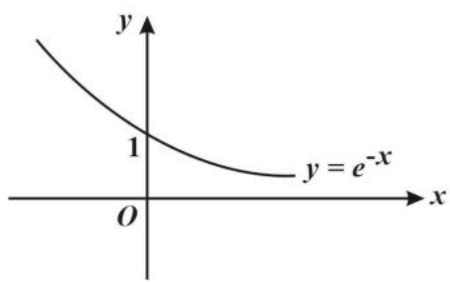
In general,

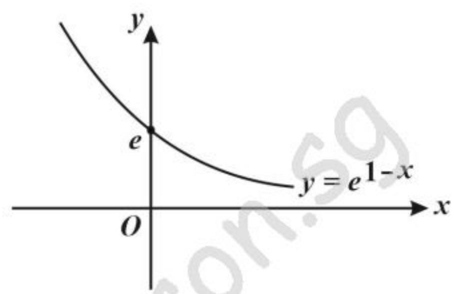
$$\frac{d}{dx}\,[\ln\,(\mathrm{f}(x))] = \frac{\mathrm{f}'(x)}{\mathrm{f}(x)}$$ where $\mathrm{f}'(x) = \frac{d}{dx}\,[\mathrm{f}(x)]$

**3. Graphs of exponential functions**

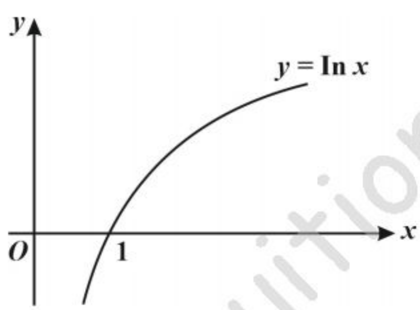


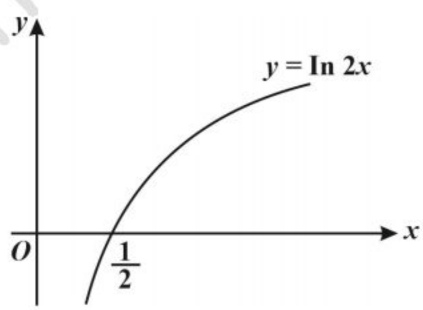




**4. Graphs of logarithmic Functions**



# TOPIC- 19

# Integration

**Revision Notes**

1. Integration is the reverse process of differentiation.

   If $\frac{dy}{dx}$ = f($x$), then $y = \int f(x)dx = g(x) + c$ where $c$ is a constant.

2. **Indefinite Integrals of some Standard Functions**

**(a) Algebraic Functions**

$$\int a\,dx = ax + c$$

$$\int x^n\,dx = \frac{x^{n+1}}{n+1} + c,\ n \neq -1$$

$$\int (ax+b)^n\,dx = \frac{(ax+b)^{n+1}}{a(n+1)} + c,\ n \neq -1,\ a \neq 0$$

$$\int \frac{1}{x}\,dx = \ln x + c,\ x > 0$$

$$\int \frac{1}{ax+b}\,dx = \frac{1}{a}\ln(ax+b) + c,\ ax+b > 0,\ a \neq 0$$

**(b) Trigonometric Functions**

$$\int \sin x\,dx = -\cos x + c$$

$$\int \cos x\,dx = \sin x + c$$

$$\int \sec^2 x\,dx = \tan x + c$$

$$\int \sin(ax+b)\,dx = -\frac{\cos(ax+b)}{a} + c$$

$$\int \cos(ax+b)\,dx = \frac{\sin(ax+b)}{a} + c$$

$$\int \sec^2 ax\,dx = \frac{\tan(ax+b)}{a} + c$$

where a ($\neq 0$), $b$ and $c$ are constants and angle $x$ is in radians.

**(c) Exponential Functions**

$\int e^x \, dx = e^x + c$

$\int e^{ax+b} \, dx = \dfrac{e^{ax+b}}{a} + c, \; a \neq 0$

where $a$, $b$ and $c$ are constants.

**3. Definite integrals**

If $\int f(x)dx = h(x) + c$, where $c$ is $a$ constant, then

$\int_a^b f(x) \, dx = [b(x)]_a^b = h(b) - h(a)$

$a$ is called the lower limit, $b$ is called the upper limit and the whole integration is called the definite integration.

**4. Some Results on Definite Integrals**

(a) $\int_b^b f(x) \, dx = 0$

(b) $\int_a^b f(x) \, dx = -\int_b^a f(x) \, dx$

(c) $\int_a^b f(x) \, dx + \int_c^b f(x) \, dx = \int_a^c f(x) \, dx$

**Worked Examples:**

***Example 1***

Integrate each of the following with respect to $x$.

(a) $\dfrac{x^4 + 7x}{x^3}$ (b) $\left(2x - \dfrac{1}{2}\right)^2$ (c) $(3x + 2)^3$

***Solution:***

(a) $\int \dfrac{x^4 + 7x}{x^3} \, dx = \int (x + 7x^{-2}) \, dx$

$= \dfrac{x^2}{2} - \dfrac{7}{x} + c$ where $c$ is a constant.

(b) $\int \left(2x - \dfrac{1}{2}\right)^2 dx = \int \left(4x^2 - 2x + \dfrac{1}{4}\right) dx$

$= \dfrac{4x^3}{3} - x^2 + \dfrac{x}{4} + c$ where $c$ is constant.

(c) $\int (3x+2)^3 \, \mathrm{dx} = \frac{(3x+2)^4}{3(4)} + c$

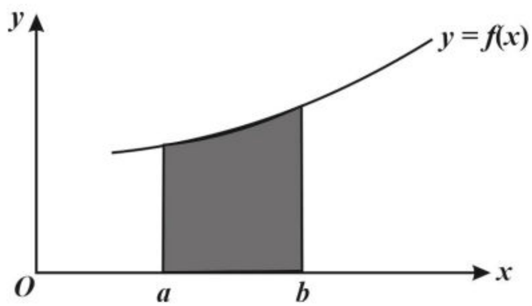
$= \frac{(3x+2)^4}{12} + c$ where $c$ is constant.

# TOPIC- 20

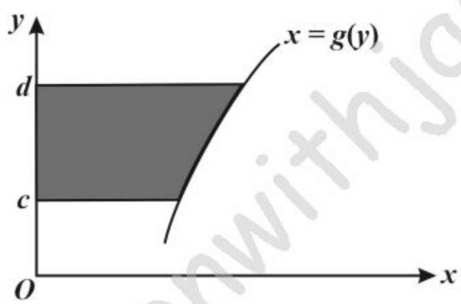
## Area of a Region

**Revision Notes**

1. The area bounded by the curve $y = f(x)$, the lines $x = a, x = b$ and the $x$ – axis is given by $\int_a^b f(x)\ dx$.




2. The area bounded by the curve $x = g(y)$, the lines $y = c, y = d$ and the $y$ – axis is given by $\int_c^d g(x)\ dy$.

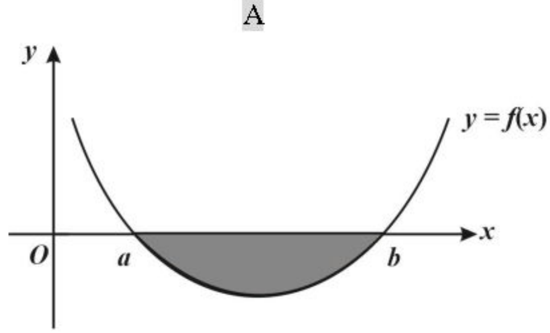


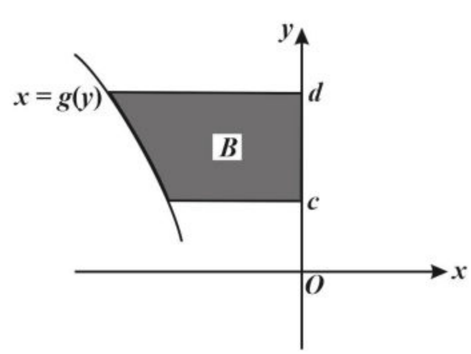

3. Area is positive when it is above the $x$ – axis or on the right of the $y$ – axis. On the reverse, the area will be negative.

Therefore, Shaded area $A = -\int_a^b f(x)\,dx$ or $\left|\int_a^b f(x)dx\right|$

Shaded area $B = -\int_c^d g(y)$ or $\left|\int_c^d g(y)dy\right|$

A



# TOPIC- 21

# Kinematics

**Revision Notes**

**1. Displacement, velocity and acceleration**

If a particle moves in a straight line with displacement s from a fixed point in time $t$, then

the velocity $v = \dfrac{ds}{dt}$.

the acceleration $a = \dfrac{dv}{dt}$.

Thus if $\dfrac{ds}{dt}$ is given, then $s = \int v\,dt$.

Similarly given , $\dfrac{dv}{dt}$ then $v = \int a\,dt$.

**Worked Examples:**

***Example 1***

A particle starts from a point $O$ and moves in a straight line so that its displacement, s. m, from $O$, $I$ seconds after saving $O$, is given by $s = I(t - 6)^2$. Obtain an expression for the velocity of the particle in terms of $r$. Hence determine the value of t when the particle first comes to instantaneous rest and find the acceleration at this instant. The particle is next at $O$ when $t = T$. find

(a) the value of $T$,

(b) the distance travelled from $t = 0$ to $t = T$.

***Solution:***

$s = \quad r\,(t - 6)^2 = t^3 - 12t^2 + 36t$

$v = \dfrac{ds}{dt} = 3t^2 - 24t + 36$

When the particle is instantaneous at rest, $v = 0$

$\Rightarrow \quad 3t^2 - 24t + 36 = 0$

$t^2 - 8t + 12 = 0$

$(t - 2)(t - 6) = 0$

$t = 2 \quad \text{or} \quad t = 6$

The particle first comes to instantaneous test when $t = 2$.

(a) When the particle is next at $O$, $s = 0$

$$\Rightarrow \text{r}\,(t-6)^2 = 0$$

$$t = 0 \text{ or } t = 6$$

(N.A.)

$$\therefore T = 6$$

(b) When $t = 2$, $s = 2(2-6)^2 = 32$

$\therefore$ The distance travelled from $t = 0$ to $t = 6 = 32 \times 2 = 64$ m

# O Level Additional Mathematics ....what they mean

## Simultaneous Equations

| | When the question ask | It means |
|---|---|---|
| 1. | Find the value of $h$ and $k$ such that the two equations<br>$3x + 2y = 4$ – Eqn (1)<br>$6x + hy = k$ – Eqn (2)<br>have no unique solutions. No unique solution also means the matrix $\begin{pmatrix} 3 & 2 \\ 6 & h \end{pmatrix}$ is singular. | For no unique solution, determinant of coefficient matrix<br>$\begin{pmatrix} 3 & 2 \\ 6 & h \end{pmatrix}$ must be equal to zero. Hence, $h = 4$<br>No unique solutions can mean either:<br>(a) Infinite Solutions<br>(b) No solutions<br>(a) For **Infinite solutions**<br>Eqn (2) **must be** a multiple of Eqn (1). Which means these 2 equations produced the same line when plotted. Hence $k = 8$<br>(b) For **No solution**<br>Eqn (2) is not a multiple of Eqn (1). Which means these 2 equations produced parallel lines when plotted. Hence $k$ can take on any real value other than 8. |

## Quadratic Equations and Inequalities

| | When the question says | It means |
|---|---|---|
| 1. | the expression $ax^2 + bx + c$ is always positive | $a > 0$ and $b^2 - 4ac < 0$ |
| 2. | the expression $ax^2 + bx + c$ is never negative | $a > 0$ and $b^2 - 4ac \leq 0$ |
| 3. | the expression $ax^2 + bx + c$ is always negative | $a < 0$ and $b^2 - 4ac < 0$ |
| 4. | the expression $ax^2 + bx + c$ is never positive | $a < 0$ and $b^2 - 4ac \leq 0$ |
| 5. | the line $y + 2x = 8$ is a tangent to the curve $y = 2x^3 - 6x + k$, | $8 - 2x = 2x^2 - 6x + k$<br>$2x^2 - 4x + k - 8 = 0$<br>and $b^2 - 4ac = 0$ |

| | When the question says | It means |
|---|---|---|
| 6. | the line $y + 2x = 8$ meets the curve $y = 2x^2 - 6x + k$, | $8 - 2x = 2x^2 - 6x + k$<br>$2x^2 - 4x + k - 8 = 0$<br>and $b^2 - 4ac \geq 0$ |
| 7. | the line $y + 2x - 8$ does not meet the curve $y = 2x^2 - 6x + k$, | $8 - 2x - 2x^2 - 6x + k$<br>$2x^2 - 4x + k - 8 = 0$<br>and $b^2 - 4ac < 0$ |
| 8. | the line $y + 2x = 8$ intersect/cuts the curve $y - 2x^2 - 6x + k$ at two distinct points. | $8 - 2x = 2x^2 - 6x + k$<br>$2x^2 - 4x + k - 8 = 0$<br>and $b^2 - 4ac > 0$ |

**Polynomials**

| | When the question says | It means |
|---|---|---|
| 1. | find the remainder when $f(x)$ is divided by $(x - a)$ | Calculate the value of $f(a)$ |
| 2. | that $f(x)$ is exactly divisible by $x^2 - 2x - 3$ | $f(3) = 0$ and $f(-1) = 0$ |

**Binomial Theorem**

| | When the question says | It means |
|---|---|---|
| 1. | Find the term independent of $x$. | Calculate the value of the constant term. |

**Trigonometric Functions**

| | When the questions are | It means |
|---|---|---|
| 1. | For the trigonometric functions :<br>(i) $a \sin bx + c$ **OR**<br>(ii) $a \cos bx + c$<br><br>(iii) $a \tan bx + c$ | Amplitude of the function is a,<br>Period of the function is $\frac{360°}{b}$ or $\frac{2\pi}{b}$<br>Maximum value of the function is $a + c$.<br>Minimum value of the function is $c - a$.<br>For tangent function, period is $\frac{180°}{b}$ or $\frac{\pi}{b}$ . There is no amplitude or maximum or minimum value as tangent function can take on any real value. |

**Differentiation**

| | **When the question says** | **It means** |
|---|---|---|
| 1. | show that $y - f(x)$ is an increasing function.... | show that $\frac{dy}{dx} > 0$ |
| 2. | show that $y = f(x)$ is a decreasing function.... | show that $\frac{dy}{dx} < 0$ |
| 3. | Find the range of values of $x$ for which $y$ is decreasing | Find the range of values of $x$ for which $\frac{dy}{dx} < 0$ |
| 4. | Find the possible values of $k$ for which the $x$–axis is a tangent to the curve $y = x^3 + 3x^2 - 9x + k$ | Step 1). Find the value of $x$ such that $\frac{dy}{dx} = 0$<br>Step 2). Substitute the value(s) of $x$ in step 1 into the equation $x^3 + 3x^2 - 9x + k = 0$ to find $k$. |

**Kinematics**

Particle is momentarily at rest $\Rightarrow v = 0$

| | **When the question says** | **It means** |
|---|---|---|
| 1. | $t$ is the time in seconds after passing a fixed point 'O' | When $t = 0$, $s = 0$ |
| 2. | $t$ is the time in seconds after the start of motion | When $t = 0$, $v = 0$ |

| | **When question asks for...** | **Steps involved:** |
|---|---|---|
| 1. | the initial displacement of the particle.<br>**[Never assume that when $t = 0$, $s = 0$]** | Find value of $a$ when $t = 0$ |
| 2. | the initial velocity/ acceleration of the particle. | Find value of $v$ or $a$ when $t = 0$ |
| 3. | the time when the particle passes through $O$ again. | Find value of $t$ when $s = 0$ |
| 4. | the time when the particle comes to instantaneous rest. | Find value of $t$ when $v = 0$ |

| | When question asks for... | Steps involved: |
|---|---|---|
| 5. | the time when the particle changes its direction of motion. | Find value of $t$ when v = 0 |
| 6. | the time interval during which the particle is travelling in the positive direction. | Find the range of values of $t$ such that<br>$v > 0$ |
| 7. | the time interval during which the particle is travelling in the negative direction. | Find the range of values of $t$ such that<br>$v < 0$ |
| 8. | the maximum displacement of the particle from the point O. | Find value($s$) of $t$ when $\frac{ds}{dt} = 0$.<br>Substitute these value($s$) of t to find the corresponding value($s$) of $s$. If more than one value, compare all the values of $s$ and the greatest value will be the maximum displacement, (positive or negative values are possible). Show that this is maximum displacement by proving that $\frac{d^2s}{dt^2} < 0$ for this value of $t$. |
| 9. | the furthest distance of the particle from the point O. | Find value($s$) of $t$ when $\frac{ds}{dt} = 0$.<br>Substitute these value($s$) of t to find the corresponding values of $\|s\|$. (Only need the absolute values)<br>If more than one value, compare all the values of $\|s\|$ and the greatest value will be the furthest distance. |
| 10. | the maximum velocity of the particle. | Find values($s$)of $t$ when $\frac{dv}{dt} = 0$<br>Substitute these value($s$) of t to find the corresponding value of v. If more than one value, compare all the values of $v$ and the greatest value will be the maximum velocity, (positive or negative values are possible). Show that this is maximum velocity by proving that $\frac{d^2v}{dt^2} < 0$ for this value of $t$. |
| 11. | the minimum velocity of the particle. | Find value($s$) of $t$ when $\frac{dv}{dt} = 0$<br>Substitute these value($s$) of t to find the corresponding value of $v$. If more than one value, compare all the values of v and the least value will be the minimum velocity, (positive or negative values are possible). Show that this is minimum velocity by proving that $\frac{d^2v}{dt^2} > 0$ for this value of $t$. |

| | When question asks for... | Steps involved: |
|---|---|---|
| 12. | the maximum speed of the particle. | Find value(*s*) of *t* when $\frac{dv}{dt} = 0$.<br>Subset these value(*s*) of *t* to find the corresponding values of $\lvert v \rvert$. (Only need the absolute values)<br>If more than one value, compare all the values of $\lvert v \rvert$ and the greatest value will be the maximum speed. |
| 13. | the speed of the particle after 4 seconds | Find value of $\lvert v \rvert$ when $t = 4$ |
| 14. | the displacement of the particle when $t = 4$. | Find value of s when $t = 4$ |
| 15. | the total distance travelled during the first 4 seconds. | Find need to know when the particle changes its direction of motion.<br>Find value of *t* when $v = 0$.<br>For example, particle changes its direction of motion at $t = a$.<br>(i) Total distance travelled<br>$= \left\lvert \int_0^a v\,dt \right\rvert + \left\lvert \int_a^4 v\,dt \right\rvert$<br>$= \left\lvert [s]_0^a \right\rvert + \left\lvert [s]_a^4 \right\rvert$ OR<br>(ii) When $t = 0$, $s = s_0$<br>When $t = a$, $s = s_n$<br>When $t = 4$, $s = s_4$<br>Total distance travelled<br>$= \lvert s_n - s_o \rvert + \lvert s_4 - s_n \rvert$ |
| 16. | the total distance travelled during the fourth second,<br>[first second is from<br>fourth second is from t = 3 to t = 4] | Total distance travelled<br>$= \left\lvert \int_3^4 v\,dt \right\rvert$<br>$= \left\lvert [s]_3^4 \right\rvert$ provided there is no turning point between<br>$t = 0$ to $t = 1$...<br>t = 3 and t = 4 |
| 17. | the time interval during which the velocity is increasing | When velocity is increasing it means the rate of change of velocity is positive.<br>Therefore, find the range of values of *t* such that<br>$\frac{dv}{dt} > 0$ |
| 18. | the time interval during which the velocity is decreasing. | When velocity is decreasing it means the rate of change of velocity is negative.<br>Therefore, find the range of values of *t* such that $\frac{dv}{dt} < 0$ |