

TOPIC-1

Simultaneous Equations

Revision Notes:

1. Simultaneous Linear and Non-Linear Equations in Two Unknowns

Use the substitution method. Express one unknown of the linear equation in terms of the other. Then we substitute this unknown in the non-linear equation to obtain a quadratic equation in one unknown which can be solved in the usual way.

TOPIC- 2

Surds, Indices and Logarithms

Revision Notes:

1. Definition of surds

When the root of a number or expression cannot be exactly determined, the root is called a SURD, e.g., $\sqrt{2}$ or $\sqrt{3}$ etc. A surd is also called an IRRATIONAL expression.

2. Operations on Surds

$$(a) \quad a\sqrt{x} + b\sqrt{x} = (a+b)\sqrt{x} \quad \text{e.g., } 2\sqrt{3} + 4\sqrt{3} = (2+4)\sqrt{3} = 6\sqrt{3}$$

$$(b) \quad a\sqrt{x} - b\sqrt{x} = (a-b)\sqrt{x} \quad \text{e.g., } 2\sqrt{3} - 4\sqrt{3} = (2-4)\sqrt{3} = -2\sqrt{3}$$

$$(c) \quad \sqrt{x} \times \sqrt{y} = \sqrt{x \times y} \quad \text{e.g., } \sqrt{3} \times \sqrt{5} = \sqrt{3 \times 5} = \sqrt{15}$$

$$(d) \quad \frac{\sqrt{x}}{\sqrt{y}} = \sqrt{\frac{x}{y}} \quad \text{e.g., } \frac{\sqrt{10}}{\sqrt{2}} = \sqrt{\frac{10}{2}} = \sqrt{5}$$

3. Rationalizing the Denominator

It is usual not to write surds in the denominator of a fraction when this can be avoided. The process of clearing irrational numbers is called rationalization.

$$(a) \quad \frac{\sqrt{x}}{\sqrt{y}} = \frac{\sqrt{x}}{\sqrt{y}} \times \frac{\sqrt{y}}{\sqrt{y}} = \frac{\sqrt{xy}}{y}$$

$$(b) \frac{1}{\sqrt{x} + \sqrt{y}} = \frac{1}{\sqrt{x} + \sqrt{y}} \times \frac{\sqrt{x} - \sqrt{y}}{\sqrt{x} - \sqrt{y}} = \frac{\sqrt{x} - \sqrt{y}}{x - y}$$

Note:

$\sqrt{x} + \sqrt{y}$ and $\sqrt{x} - \sqrt{y}$ are called conjugate surds. Their product, $(\sqrt{x} + \sqrt{y})(\sqrt{x} - \sqrt{y}) = x - y$, is a rational number.

4. Law of Indices

If a and b are positive and m and n , any rational indices, then

$$(a) a^m \times a^n = a^{m+n}$$

$$(b) a^m \div a^n = a^{m-n}$$

$$(c) (a^m)^n = a^{mn}$$

$$(d) a^m \times b^m = (ab)^m$$

$$(e) a^m \div b^m = \left(\frac{a}{b}\right)^m$$

$$(f) a^0 = 1, a \neq 0$$

$$(g) a^{-n} = \frac{1}{a^n}$$

$$(h) a^{\frac{1}{n}} = \sqrt[n]{a}$$

$$(i) a^{\frac{m}{n}} = \sqrt[n]{a^m} = (\sqrt[n]{a})^m$$

Note:

When $a > 0$, $a^x > 0$ for all real values of x .

5. Logarithms

If $a^x = b$, $a > 0$ (Index form)

Then $\log_a b = x$ (Logarithmic form)

The logarithm to the base 10 is known as common logarithm.

The logarithm to the base e (≈ 2.718) is known as the natural logarithm.

Note: $\log_{10} x$ is also written as $\lg x$.

$\log_e x$ is also written as $\ln x$.

6. Laws of Logarithms

$$(a) \log_a xy = \log_a x + \log_a y$$

$$(b) \log_a \frac{x}{y} = \log_a x - \log_a y$$

- (c) $\log_a x^n = n \log_a x$
 (d) $\log_a 1 = 0$
 (e) $\log_a a = 1$
 (f) $\log_a x = \frac{\log_b x}{\log_b a}$, $\log_a x = \frac{1}{\log_x a}$ (change of base)

Note: $\log_a x$ exists if $a > 0$ and $x > 0$.

Worked Examples:

Example 1

Simplify the following.

(a) $\sqrt{50} + \sqrt{2} - 2\sqrt{18} + \sqrt{8}$ (b) $(2 + \sqrt{3})^2 - (3 - \sqrt{3})^2$

Solution:

(a) $\sqrt{50} + \sqrt{2} - 2\sqrt{18} + \sqrt{8} = \sqrt{25 \times 2} + \sqrt{2} - 2\sqrt{9 \times 2} + \sqrt{4 \times 2}$
 $= 5\sqrt{2} + \sqrt{2} - 6\sqrt{2} + 2\sqrt{2}$
 $= 2\sqrt{2}$
 (b) $(2 + \sqrt{3})^2 - (3 - \sqrt{3})^2 = 2^2 + 2(2)(\sqrt{3}) + (\sqrt{3})^2 - [3^2 - 2(3)(\sqrt{3}) + (\sqrt{3})^2]$
 $= 4 + 4\sqrt{3} + 3 - [9 - 6\sqrt{3} + 3]$
 $= 7 + 4\sqrt{3} - [12 - 6\sqrt{3}]$
 $= 7 + 4\sqrt{3} - 12 + 6\sqrt{3}$
 $= 10\sqrt{3} - 5$

TOPIC- 3

Quadratic Equations and Inequalities

Revision Notes:

Nature of Roots of a Quadratic Equation

Consider the quadratic equation $ax^2 + bx + c = 0$ where a , b and c are constants and $a \neq 0$.

The following formula can be used to solve the quadratic equation

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

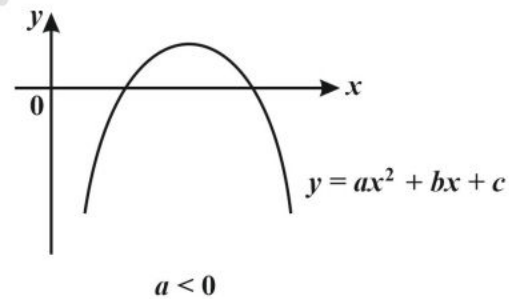
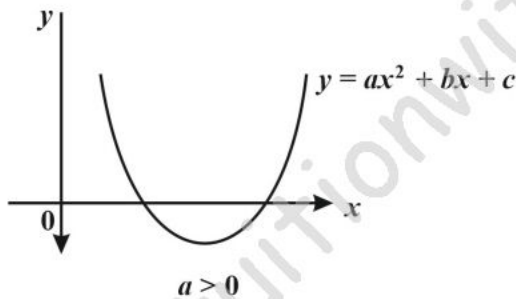
Types of Roots

From the above formula, we find that, in general, there are two roots for every quadratic equation, but the two roots may be unequal or equal.

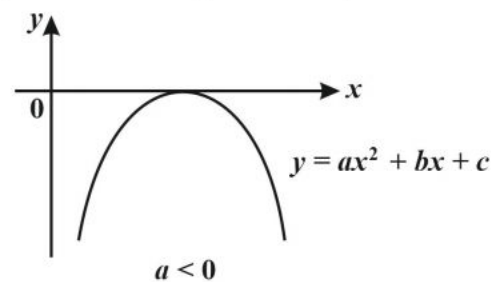
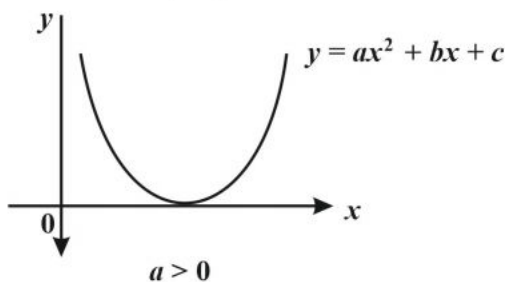
The roots of the equation will depend upon the value of $b^2 - 4ac$. The value $b^2 - 4ac$ is called the discriminant, of the quadratic equation $ax^2 + bx + c = 0$.

There are three types of roots of the quadratic equation

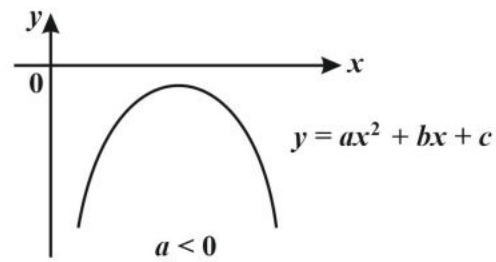
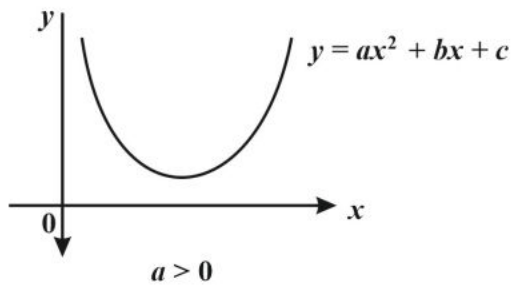
- (a) If $b^2 - 4ac > 0$, the two roots are real and distinct (or 2 real unequal roots)



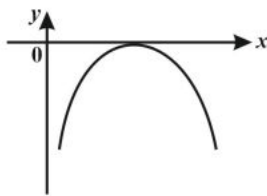
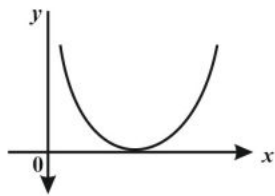
- (b) If $b^2 - 4ac = 0$, the two roots are real and equal (or coincident / repeated roots)



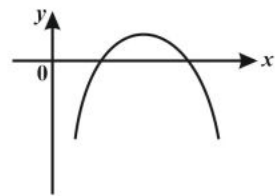
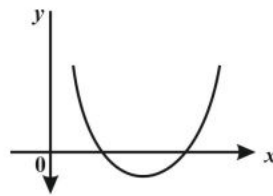
- (c) If $b^2 - 4ac < 0$, the two roots are complex or imaginary (i.e., no real roots)



Note: If $b^2 - 4ac \geq 0$, the two roots are real (i.e., the roots are real and equal or real and unequal)



or



3. Conditions for the Intersection of a Straight Line and a Curve

If the equation of a straight line is $y = mx + c$ and the equation of a curve is $y = ax^2 + bx + d$, where a, b, c, d and m are constants.

At the point of intersection,

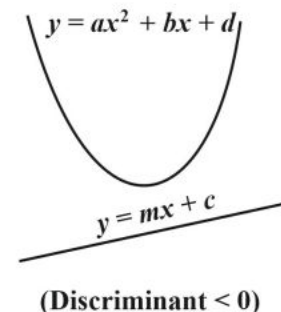
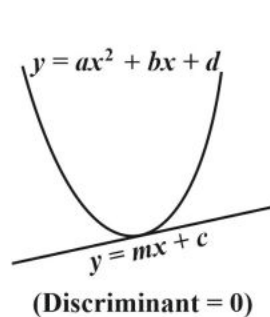
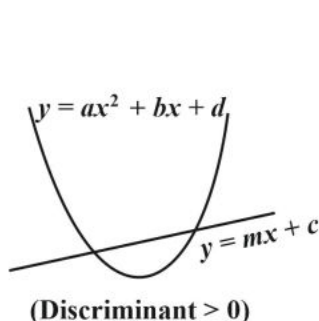
$$ax^2 + bx + d = mx + c$$

$$ax^2 + (b - m)x + (d - c) = 0 \quad (1)$$

The solution from the equation (1) will give the x -coordinate of the points of intersection of these two graphs.

In equation (1), if

- (i) the **discriminant** > 0 , then the line intersects the curve at two distinct points,
- (ii) the **discriminant** $= 0$, then the line will only meet the curve at one point and the line is a tangent to the curve,
- (iii) the **discriminant** < 0 , then the line does not meet the curve.

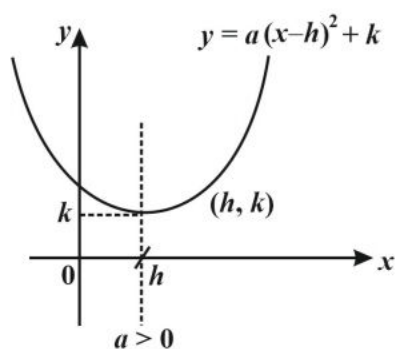


4. Maximum / Minimum Value of a Quadratic Function

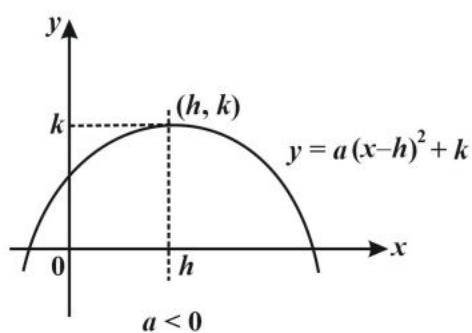
We can express a given quadratic function $f(x) = ax^2 + bx + c$ in the form of $f(x) = a(x - h)^2 + k$ using the completing square method.

A quadratic function expressed in the form of $a(x - h)^2 + k$ has a maximum or minimum value

(depending on the value of a) of k when $x = h$.



Minimum value = k
when $x = h$
Minimum point (h, k)



Minimum value = k
when $x = h$
Minimum point (h, k)

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TOPIC - 4

Partial Fractions

Revision Notes:

For two polynomials in x , $N(x)$ and $D(x)$, the rational function $\frac{N(x)}{D(x)}$ is a proper fraction if the degree of $N(x)$ is strictly less than that of $D(x)$. Otherwise, it is an improper fraction.

e.g., $\frac{1}{x^2-4}$, $\frac{2x^2+3}{x(x-1)^2}$ and $\frac{1}{x(x^2+x+1)}$ are proper fractions

$\frac{x^5}{x^2-1}$ is an improper fraction

An improper rational function can be converted, by division or otherwise, into a sum of polynomial and a proper fraction.

e.g., $\frac{x^5}{x^2-1} = x^3 + x + \frac{x}{x^2-1}$

$$\begin{array}{r} x^3 + x \\ x^2 - 1 \overline{) x^5} \\ \underline{x^5 - x^3} \\ x^3 - x \\ \underline{x^3 - x} \\ 0 \end{array}$$

A single rational function can be expressed as a sum of two or more simpler fractions, called partial fractions.

To express $\frac{N(x)}{D(x)}$ in partial fractions:

Step 1: Check whether $\frac{N(x)}{D(x)}$ is proper. If not, express it as a sum of polynomial and a proper fraction

using long division or manipulate the numerator so that simple cancellation can be done.

Step 2: Check if the denominator is completely factorised.

Step 3: Express in partial fractions using cases 1, 2 and 3.

Case	Denominator of fraction with	Algebraic fraction	Expression used
1	Linear factor	$\frac{mx+n}{(ax+b)(cx+d)}$	$= \frac{A}{ax+b} + \frac{C}{cx+d}$
2	Repeated factors	$\frac{mx+n}{(ax+b)(cx+d)^2}$	$= \frac{A}{ax+b} + \frac{B}{cx+d} + \frac{C}{(cx+d)^2}$
3	Quadratic factor which does not factorise	$\frac{mx+n}{(ax+b)(x^2+c^2)}$	$= \frac{A}{ax+b} + \frac{Bx+C}{x^2+c^2}$

Step 4: Solve for unknown constants by

- (i) substituting values of x and/or
- (ii) comparing coefficients of like terms.

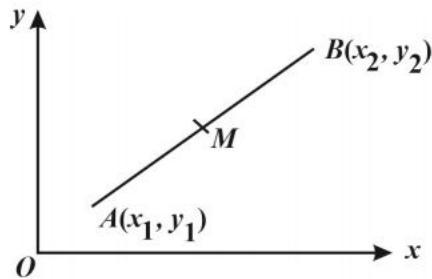
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TOPIC-7

Coordinate Geometry

Revision Notes:

1. Distance and mid-point

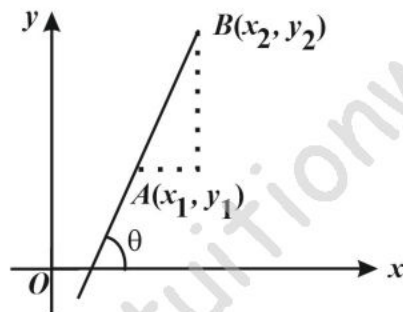


The distance AB between two points $A(x_1, y_1)$ and $B(x_2, y_2)$ is $\sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$.

The mid-point M of $A(x_1, y_1)$ and $B(x_2, y_2)$ is $\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$

2. Gradient of a straight line

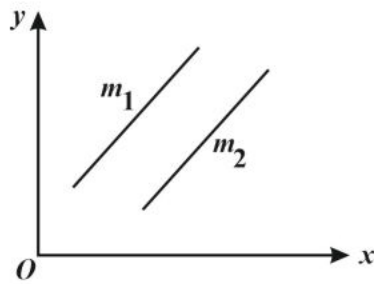
Definition



Gradient of a line is defined as the tangent of the angle which the line makes with the positive direction of the x -axis.

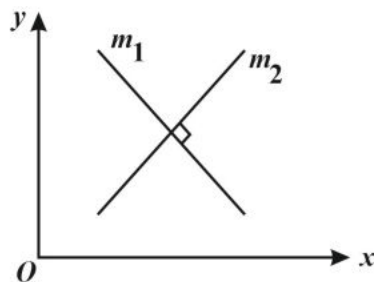
Given $A(x_1, y_1)$ and $B(x_2, y_2)$ then the gradient of AB is $\frac{y_2 - y_1}{x_2 - x_1} = \tan \theta$.

Gradient of parallel lines



The gradient of a parallel lines are equal, i.e., $m_1 = m_2$.

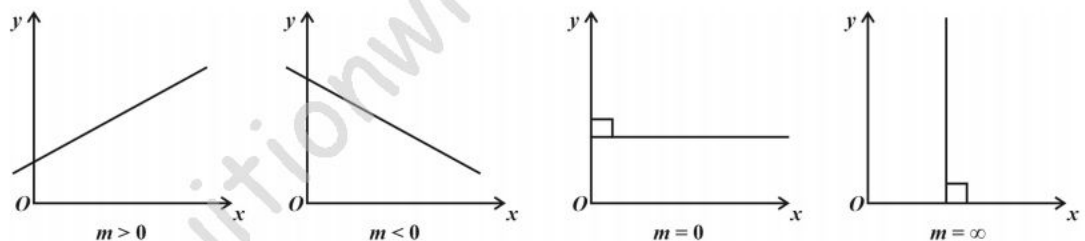
Gradients of perpendicular lines



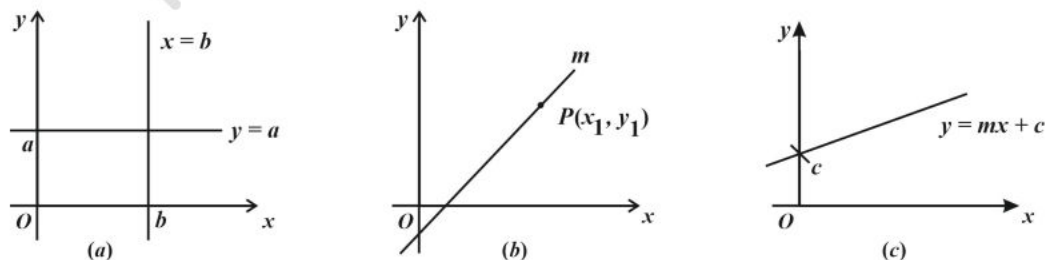
The product of the gradients of two perpendicular lines is -1 .

i.e., $m_1 \times m_2 = -1$ or $m_1 = \frac{1}{m_2}$

Gradients of different types of straight lines



3. The straight lines



(a) Lines parallel to the coordinates axes

Equation of straight line parallel to the x -axis is $y = a$ where a is a constant

Equation of straight line parallel to the y -axis is $x = b$ where b is a constant

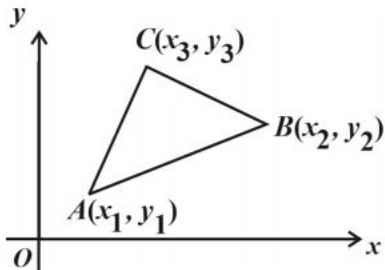
(b) Straight line of gradient and point form

If a straight line passing through a point $P(x_1, y_1)$ and having a gradient m , then its equation is $y - y_1 = m(x - x_1)$.

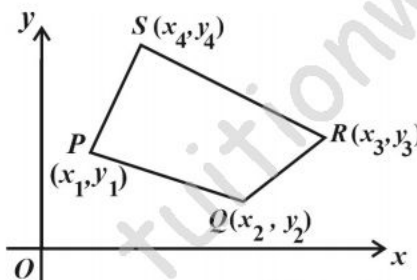
(c) Gradient form of straight line

If a straight line makes a y -intercept of length c and having a gradient m , then its equation is $y = mx + c$.

4. Area of Plane Figures (Given the vertices)

(a) 

$$\begin{aligned} \text{Area of } \triangle ABC &= \frac{1}{2} \left| \begin{array}{cccc} x_1 & x_2 & x_3 & x_1 \\ y_1 & y_2 & y_3 & y_1 \end{array} \right| \\ &= \frac{1}{2} | [(x_1y_2 + x_2y_3 + x_3y_1) - (y_1x_2 + y_2x_3 + y_3x_1)] | \end{aligned}$$

(b) 

$$\begin{aligned} \text{Area of } \triangle PQRS &= \frac{1}{2} \left| \begin{array}{ccccc} x_1 & x_2 & x_3 & x_4 & x_1 \\ y_1 & y_2 & y_3 & y_4 & y_1 \end{array} \right| \\ &= \frac{1}{2} | [(x_1y_2 + x_2y_3 + x_3y_4 + x_4y_1) - (y_1x_2 + y_2x_3 + y_3x_4 + y_4x_1)] | \end{aligned}$$

Note: When the vertices are listed in anticlockwise direction, the area is positive. If the vertices are listed in clockwise direction, the area is negative. Discard the negative sign.

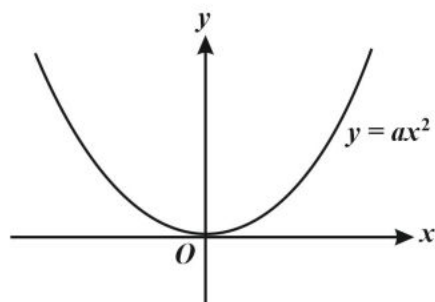
TOPIC - 9

Further Coordinate Geometry

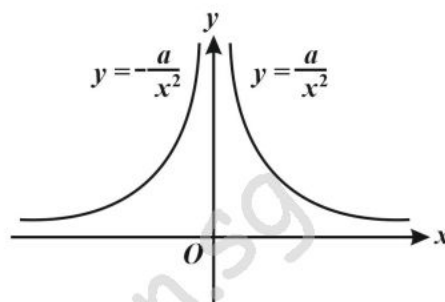
Revision Notes:

1. Graphs of the form $y = ax^n$, $a > 0$

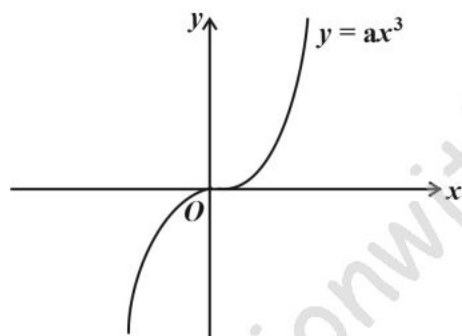
(a) $n = 2$



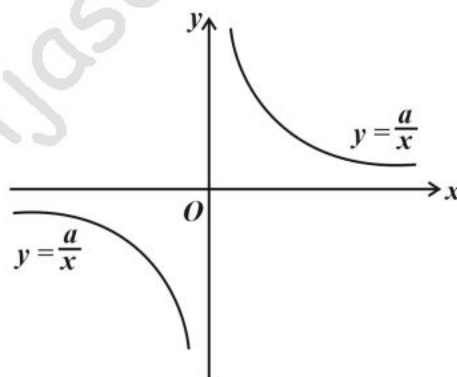
(b) $n = -2$



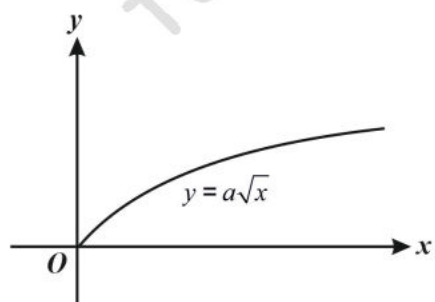
(c) $n = 3$



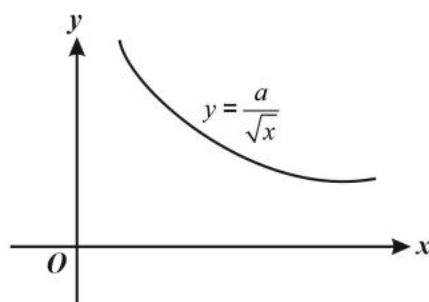
(d) $n = -1$



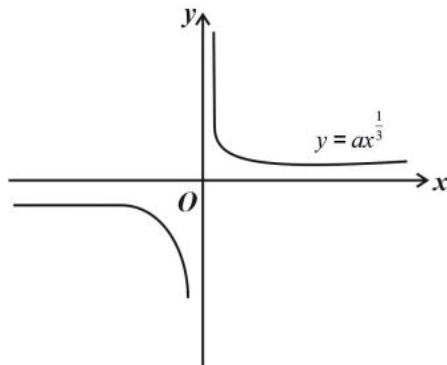
(e) $n = \frac{1}{2}$



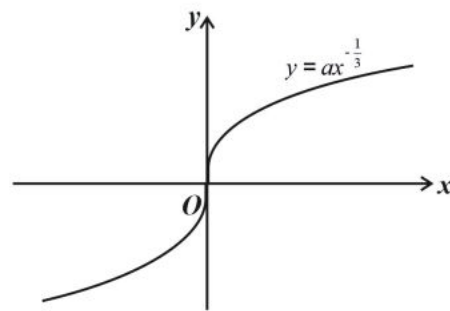
(f) $n = -\frac{1}{2}$



(g) $n = \frac{1}{3}$

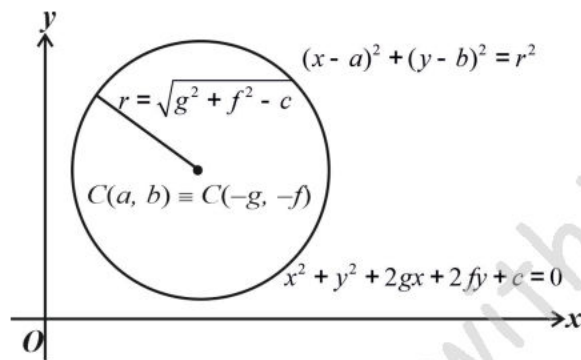


(h) $n = -\frac{1}{3}$



Note: The graphs are symmetrical about the x -axis.

Circles



Centre: $C(a, b) = C(g, f)$

Radius: $r = \sqrt{g^2 + f^2 - c}$

Equation: $(x - h)^2 + (y - k)^2 = r^2$

$x^2 + y^2 + 2gx + 2fy + c = 0$

Worked Examples:

Example 1

Find the centre and radius of the circle $x^2 + y^2 + 2x - 4y - 20 = 0$.

Find the equation of the tangent at $P(2, 6)$.

Solution:

$$x^2 + y^2 + 2x - 4y - 20 = 0$$

$$(x_T + 1)^2 - 1 + (y - 2)^2 - 4 - 20 = 0$$

$$(x+1)^2 + (y - 2)^2 = 25$$

Centre is $(-1, 2)$ and radius is 5 units.

TOPIC- 11

Remainder and Factor Theorem

Revision Notes

1. Remainder Theorem

If a polynomial $f(x)$ is divided by $(x - a)$, the remainder is $f(a)$

2. Factor Theorem

If a polynomial $f(x)$ is divided by $(x - a)$ and the remainder $f(a) = 0$, then $(x - a)$ is a factor of $f(x)$.
Conversely, if $(x - a)$ is a factor of $f(x)$, then $f(a) = 0$.

3. Solving Cubic Equations

To find the solution of a given cubic equation, $ax^3 + bx^2 + cx + d = 0$, where a, b, c and d are constant

Step 1: Find one factor by trial and error using the Factor Theorem.

Step 2: Use either **Long Division** or the **Inspection** method to find the other factors.

Step 3: Solve the equations for the roots.

4. Identity

A mathematical identity is a relation which is true for all values of the unknown.

e.g. $(x + 1)^2 = x^2 + 2x + 1$ for all values of x .

Worked Examples:

Example 1

The expression $2x^3 + ax^2 - 3x + b$ is exactly divisible by $x^2 + x - 2$. Find the value of a and of b .

Solution:

$$\text{Let } f(x) = 2x^3 + ax^2 - 3x + b$$

$$x^2 + x - 2 = (x + 2)(x - 1)$$

$f(x)$ is exactly divisible by $x^2 + x - 2 \Rightarrow (x + 2)$ and $(x - 1)$ are factors of $f(x)$.

$$f(-2) = 0$$

$$f(-2) = 0$$

$$2(-2)^3 + a(-2)^2 - 3(-2) + b = 0$$

$$-16 + 4a + 6 + b = 0$$

$$4a + b = 10 \quad (1)$$

$$f(1) = 0$$

$$2(1)^3 + a(1)^2 - 3(1) + b = 0$$

$$2 + a - 3 + b = 0$$

$$a + b = 1 \quad (2)$$

$$(1) - (2) \quad 3a = 9$$

$$a = 3$$

$$\text{Sub, } a = 3 \text{ into (2), } b = 1 - 3 = -2$$

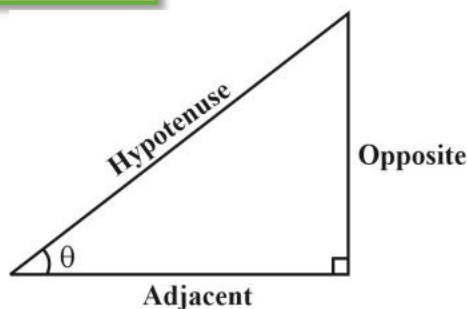
$$\therefore a = 3, b = -2$$

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TOPIC- 12

Trigonometry

Revision Notes:



$$\sin \theta = \frac{\text{Opposite}}{\text{Hypotenuse}}$$

$$\cos \theta = \frac{\text{Adjacent}}{\text{Hypotenuse}}$$

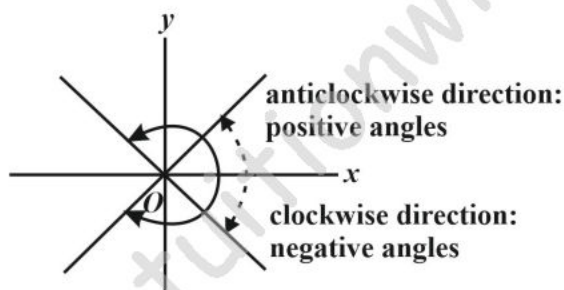
$$\tan \theta = \frac{\text{Opposite}}{\text{Adjacent}}$$

1. Relation between Radian and Degree

$$\pi \text{ radians} = 180^\circ$$

$$\Rightarrow 1 \text{ radian} = \frac{180^\circ}{\pi}, 1^\circ = \frac{\pi}{180^\circ} \text{ radian}$$

2. Positive and Negative Angles



Angle measured from ox in anticlockwise sense is positive angle.

Angle measured from ox in clockwise sense is negative angle.

3. Signs of Trigonometric Ratios in the Four Quadrants

2nd quadrant	1st quadrant
Sine Positive	All Positive
Tangent Positive	Cosine Positive
3rd quadrant	4th quadrant

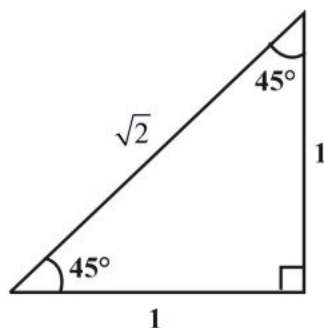
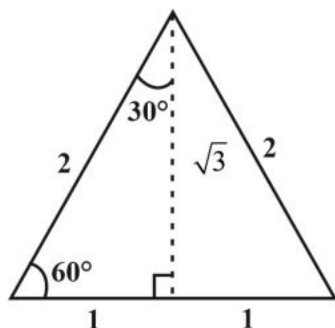
Note: To remember the positive ratio use

S	A
T	C

4. Trigonometrical Ratio of Special Angles

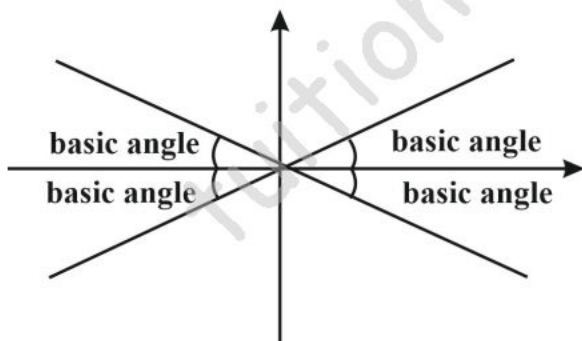
For 30° or 60° , consider an equilateral triangle of side 2 units.

For 45° , consider a right-angled isosceles triangle.



θ	30°	45°	60°
$\sin \theta$	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{\sqrt{3}}{2}$
$\cos \theta$	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$
$\tan \theta$	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$

6. The basic angle is the acute angle between a rotating radius about the origin and the x -axis



7. Trigonometrical Identities

(a) $\tan \theta = \frac{\sin \theta}{\cos \theta}$

(b) $\sin(-\theta) = -\sin \theta$

(c) $\sin(90^\circ - \theta) = \cos \theta$

$\cot \theta = \frac{\cos \theta}{\sin \theta} = \frac{1}{\tan \theta}$

$\cos(-\theta) = \cos \theta$

$\cos(90^\circ - \theta) = \sin \theta$

$$\sec \theta = \frac{1}{\cos \theta}$$

$$\tan (-\theta) = -\tan \theta$$

$$\tan (90^\circ - \theta) = \cot \theta = \frac{1}{\tan \theta}$$

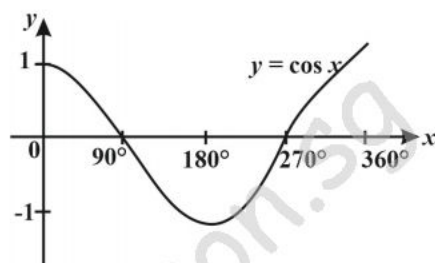
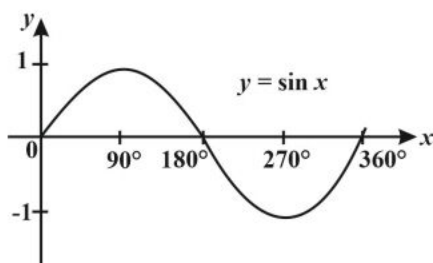
$$\operatorname{cosec} \theta = \frac{1}{\sin \theta}$$

$$(d) \sin (180^\circ - \theta) = \sin \theta \quad (e) \sin^2 \theta + \cos^2 \theta = 1$$

$$\cos (180^\circ - \theta) = -\cos \theta \quad 1 + \tan^2 \theta = \sec^2 \theta$$

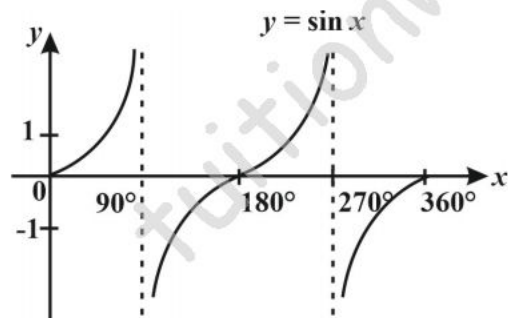
$$\tan (180^\circ - \theta) = -\tan \theta \quad 1 + \cot^2 \theta = \operatorname{cosec}^2 \theta$$

8. Graphs of $\sin x$, $\cos x$ and $\tan x$



From the above graphs, we note the following:

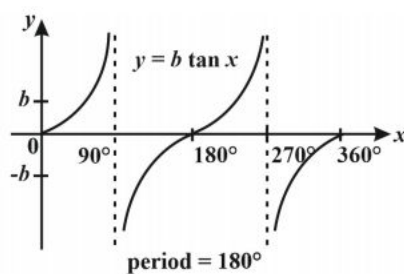
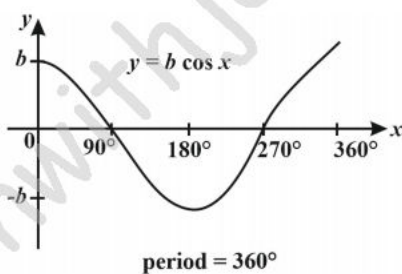
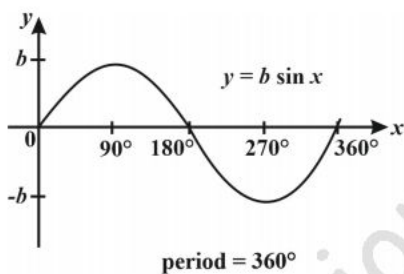
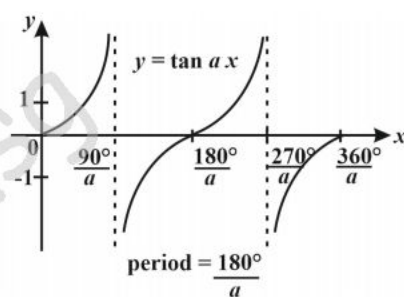
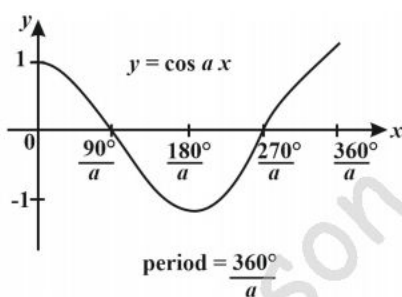
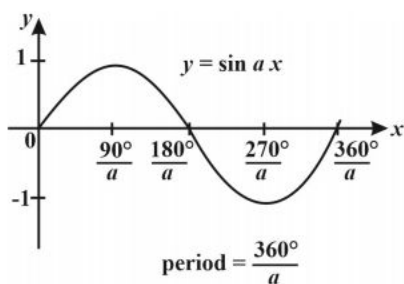
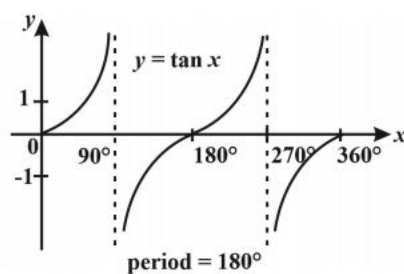
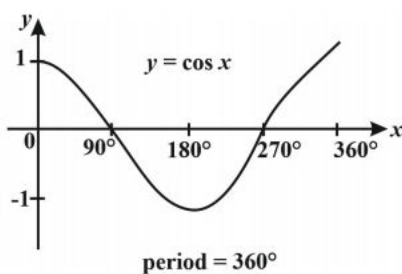
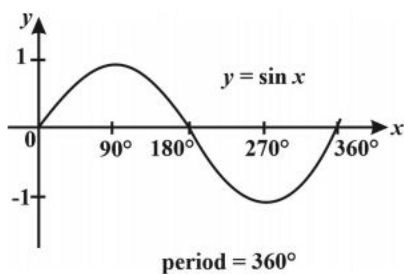
- (a) The graphs of $\sin x$ and $\cos x$ will repeat itself after an interval 360° or 2π ;
i.e., the period = 360° or 2π .
- (b) The maximum value and minimum values of $\sin x$ and $\cos x$ are 1 and -1 respectively.
i.e., the magnitude or amplitude = 1.



From the graph of $y = \tan x$, we note the following:

- (a) $\tan x$ is periodic with period 180° or π .
- (b) $\tan x$ has no maximum or minimum value. It can take all real values.
- (c) $\tan x$ is undefined when $x = 90^\circ \left(= \frac{\pi}{2} \right)$ or $270^\circ \left(= \frac{3\pi}{2} \right)$. The approach the vertical line drawn through $x = 90^\circ, 270^\circ$. Such that vertical lines through $x = 90^\circ$ and 270° are called the asymptotes of the graph.

9. More Trigonometrical Graphs



$$= a \sin bx + c \quad \text{period} = \frac{360^\circ}{b} \left(\text{or } \frac{2\pi}{b} \right) \quad \text{amplitude} = a$$

$$= a \cos bx + c \quad \text{period} = \frac{360^\circ}{b} \left(\text{or } \frac{2\pi}{b} \right) \quad \text{amplitude} = a$$

$$= a \tan bx + c \quad \text{period} = \frac{180^\circ}{b} \left(\text{or } \frac{\pi}{b} \right)$$

10. Principal Values

$$-90^\circ \leq \sin^{-1} y \leq 90^\circ \left(\text{or } -\frac{\pi}{2} \leq \sin^{-1} y \leq \frac{\pi}{2} \right)$$

$$0^\circ \leq \cos^{-1} y \leq 180^\circ \left(\text{or } 0 \leq \cos^{-1} y \leq \pi \right)$$

$$-90^\circ < \tan^{-1} y < 90^\circ \left(\text{or } -\frac{\pi}{2} < \tan^{-1} y < \frac{\pi}{2} \right)$$

TOPIC - 13

Further Trigonometry

Revision Notes

1. Addition Formulae

$$\sin (A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos (A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\tan (A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$

2. Double Angle Formulae

$$\sin 2A = 2 \sin A \cos A$$

$$\cos 2A = \cos^2 A - \sin^2 A$$

$$= 2 \cos^2 A - 1$$

$$= 1 - 2 \sin^2 A$$

$$\tan 2A = \frac{2 \tan A}{1 - \tan^2 A}$$

3. R-Formulae

$$a \sin x \pm b \cos x = R \sin (x \pm a)$$

$$a \cos x \pm b \sin x = R \cos (x \mp a)$$

$$\text{where } R = \sqrt{a^2 + b^2} \text{ and } \tan a = \frac{b}{a}$$

TOPIC - 14

Differentiation

Revision Notes

1. If $y = f(x)$ represents a line or curve, $\frac{dy}{dx}$ represents the gradient of the line or the curve at a point.

The gradients of the curve at different points have different gradients.

2. $\frac{dy}{dx}$ or $f'(x)$ is also called the **first derivative**, the **derived function** or the **differential coefficient** of y with respect to x .

3. $\frac{d^2y}{dx^2}$ is called the *second derivative*,

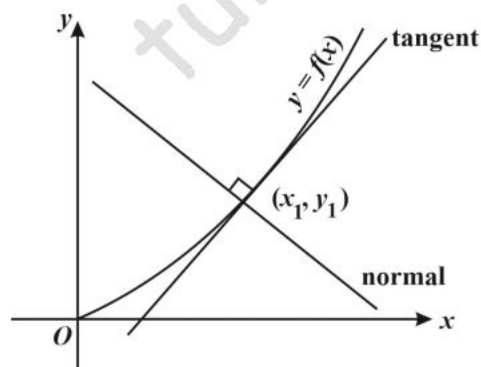
4. If $y = k$, where k is a constant, $\frac{dy}{dx} = 0$

5. If $y = kx^n$, $\frac{dy}{dx} = knx^{n-1}$

6. If $y = u \pm v$ where $u = f(x)$, $v = g(x)$, $\frac{dy}{dx} = \frac{du}{dx} \pm \frac{dv}{dx}$

7. If $y = f(u)$, $u = g(x)$, $\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx}$ (Chain Rule)

$$\frac{dy}{dx} = \frac{1}{\frac{dx}{dy}}$$



Equation of the tangent: $y - y_1 = m(x - x_1)$

Equation of the normal: $y - y_1 = -\frac{1}{m}(x - x_1)$ where $m = \frac{dy}{dx}$ at the point (x_1, y_1)

8. Differentiation of a product (Product rule)

If $y = uv$ where $u = f(x)$ and $v = g(x)$, then $\frac{dy}{dx} = u \frac{dv}{dx} + v \frac{du}{dx}$

9. Differentiation of a quotient (Quotient rule)

If $y = \frac{u}{v}$ where $u = f(x)$ and $v = g(x)$, then $\frac{dy}{dx} = \frac{v \frac{du}{dx} - u \frac{dv}{dx}}{v^2}$

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TOPIC- 15

Rate of Change

Revision Notes

$\frac{dy}{dx}$ is the rate of change of y with respect to x .

If x and y are given in terms of the time, the rates of change $\frac{dx}{dt}$ and $\frac{dy}{dt}$ are related by:

$$\frac{dy}{dx} = \frac{dy}{dt} \times \frac{dt}{dx}$$

Worked Examples:

Example 1

A viscous liquid is poured on to a flat surface. It forms a circular patch which grows at a steady rate of $5 \text{ cm}^2/\text{s}$. Find, in terms of π ,

- the radius of the patch 20 seconds after pouring has commenced.
- the rate of increase of the radius at this instant

Solution:

- $A = \pi r^2$ where r = radius of circle

A = area of circle

After 20 s, $A = 5 \times 20 = 100$

$$\pi r^2 = 100$$

$$r^2 = \frac{100}{\pi}$$

$$r = \frac{10}{\sqrt{\pi}}$$

\therefore The radius of the patch is $\frac{10}{\sqrt{\pi}} \text{ cm}$.

- $\frac{dA}{dr} = 2\pi r$

When $r = \frac{10}{\sqrt{\pi}}$, $\frac{dA}{dr} = 2\pi \left(\frac{10}{\sqrt{\pi}} \right) = 20\sqrt{\pi}$

$$\begin{aligned}\frac{dr}{dt} &= \frac{dr}{dA} \times \frac{dA}{dr} \\ &= \frac{1}{20\sqrt{\pi}} \times 5 \\ &= \frac{1}{4\sqrt{\pi}}\end{aligned}$$

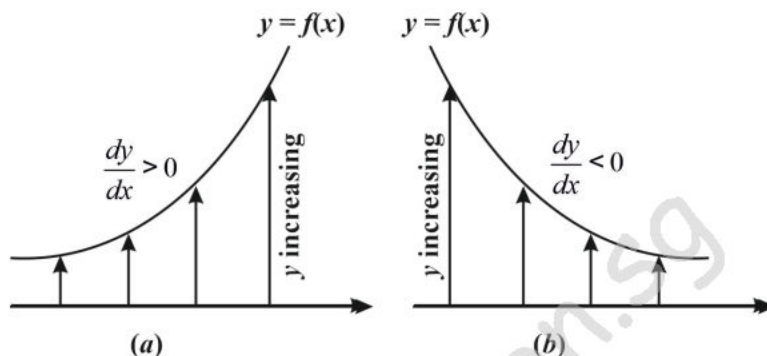
\therefore The rate of increase of the radius is $\frac{1}{4\sqrt{\pi}}$ cm/s.

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TOPIC-16

Stationary Points, Maxima and Minima Problems

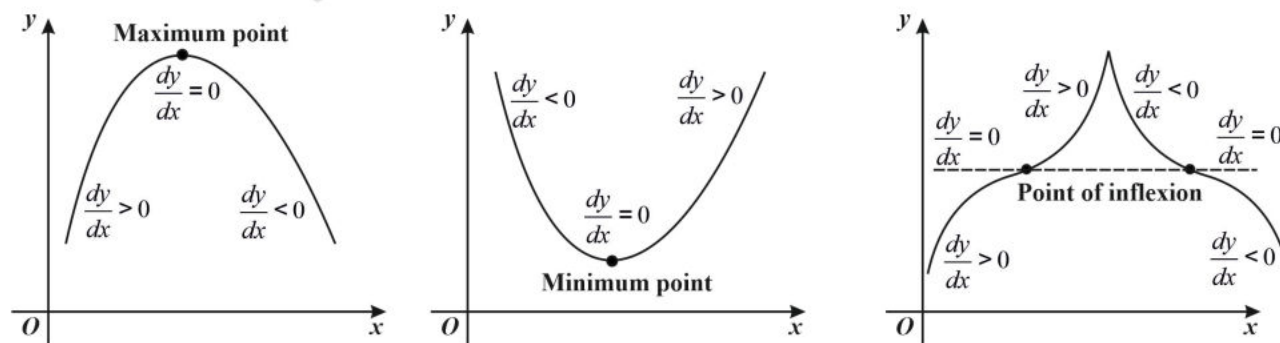
Revision Notes:



- In Fig. (a), on any stretch of the curve $y = f(x)$ where $\frac{dy}{dx} > 0$, the curve slopes upwards. Hence y increases as x increases.

Similarly, if $\frac{dy}{dx} < 0$ (Fig. (b)), y decrease as x increases.

- A stationary point on a curve is defined as a point where the $\frac{dy}{dx} = 0$.
- There are two types of stationary points:
 - turning points (maximum / minimum points)
 - points of inflexion



4. Test for a maximum point, minimum point, point of inflexion

	1st derivative test	2nd derivative test
Maximum point	changes from positive to negative	$\frac{d^2y}{dx^2} < 0$
Minimum point	changes from negative to positive	$\frac{d^2y}{dx^2} > 0$
Point of inflexion	does not change sign	—

Note: Maximum and minimum applies around the stationary point. The values of the function at this point are not necessarily the greatest and least values of the whole function.

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TOPIC -17

Derivatives of Trigonometric Functions

Revision Notes:

$$1. \quad \frac{d}{dx} (\sin x) = \cos x$$

$$\frac{d}{dx} (\cos x) = -\sin x$$

$$\frac{d}{dx} (\tan x) = \sec^2 x$$

$$2. \quad \frac{d}{dx} [\sin (ax + b)] = a \cos (ax + b)$$

$$\frac{d}{dx} [\cos (ax + b)] = -a \sin (ax + b)$$

$$\frac{d}{dx} [\tan (ax + b)] = a \sec^2 (ax + b)$$

where a and b are constants.

$$3. \quad \frac{d}{dx} (\sin^n x) = n \sin^{n-1} x \cos x$$

$$\frac{d}{dx} (\cos^n x) = -n \cos^{n-1} x \sin x$$

$$\frac{d}{dx} (\tan^n x) = n \tan^{n-1} x \sec^2 x$$

Worked Examples:

Example 1

Differentiate the following with respect to x .

(a) $\sin^3 2x$

(b) $\sqrt{1 + 3 \cos^2 x}$

Solution:

$$(a) \quad \frac{d}{dx} (\sin^3 2x) = 3(\sin^2 2x) (\cos 2x) (2)$$

$$= 6 \sin^2 2x \cos 2x$$

$$(b) \quad \frac{d}{dx} \sqrt{1+3\cos^2 x} = \frac{d}{dx} (1+3\cos^2 x)^{\frac{1}{2}}$$

$$= \frac{1}{2} (1+3\cos^2 x)^{-\frac{1}{2}} (6\cos x) (-\sin x)$$

$$= \frac{-3\sin x \cos x}{\sqrt{1+3\cos^2 x}}$$

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TOPIC- 18

Exponential and Logarithmic Functions

Revision Notes:

1. Differentiation of Exponential Functions

$$\frac{d}{dx} e^x = e^x$$

$$\frac{d}{dx} (e^{ax+b}) = ae^{ax+b} \quad \text{where } a \text{ and } b \text{ are constants}$$

In general,

$$\frac{d}{dx} (e^{f(x)}) = e^{f(x)} \times \frac{d}{dx} [f(x)]$$

2. Differentiation of Logarithmic Functions

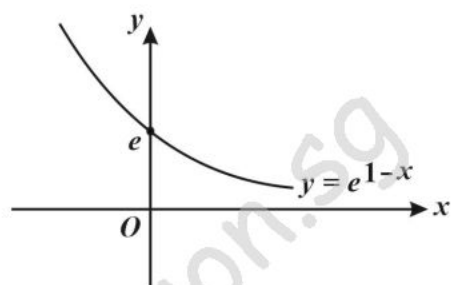
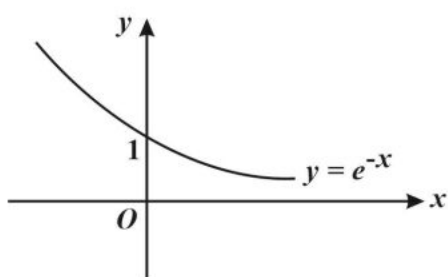
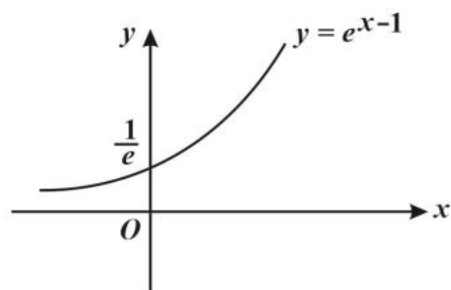
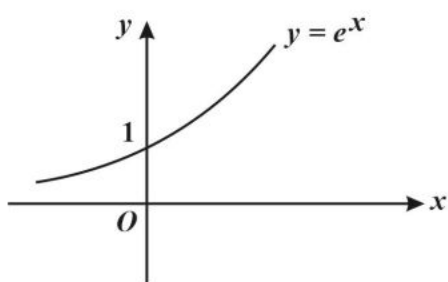
$$\frac{d}{dx} (\ln x) = \frac{1}{x}$$

$$\frac{d}{dx} [\ln (ax + b)] = \frac{a}{ax + b} \quad \text{where } a \text{ and } b \text{ are constants}$$

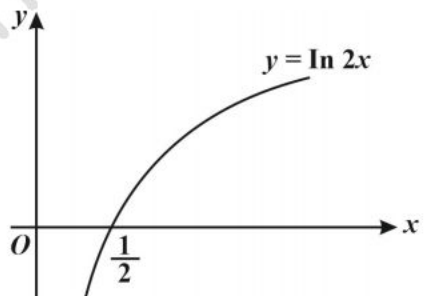
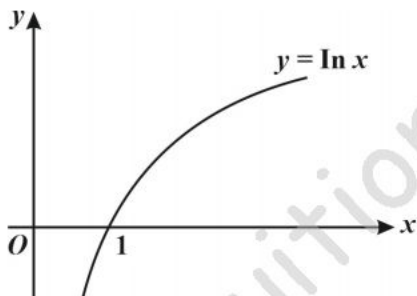
In general,

$$\frac{d}{dx} [\ln (f(x))] = \frac{f'(x)}{f(x)} \quad \text{where } f'(x) = \frac{d}{dx} [f(x)]$$

3. Graphs of exponential functions



4. Graphs of logarithmic Functions



TOPIC- 19

Integration

Revision Notes

- Integration is the reverse process of differentiation.

If $\frac{dy}{dx} = f(x)$, then $y = \int f(x)dx = g(x) + c$ where c is a constant.

- Indefinite Integrals of some Standard Functions**

(a) Algebraic Functions

$$\int a \, dx = ax + c$$

$$\int x^n \, dx = \frac{x^{n+1}}{n+1} + c, n \neq -1$$

$$\int (ax + b)^n \, dx = \frac{(ax + b)^{n+1}}{a(n+1)} + c, n \neq -1, a \neq 0$$

$$\int \frac{1}{x} \, dx = \ln x + c, x > 0$$

$$\int \frac{1}{ax + b} \, dx = \frac{1}{a} \ln(ax + b) + c, ax + b > 0, a \neq 0$$

(b) Trigonometric Functions

$$\int \sin x \, dx = -\cos x + c$$

$$\int \cos x \, dx = \sin x + c$$

$$\int \sec^2 x \, dx = \tan x + c$$

$$\int \sin(ax + b) \, dx = -\frac{\cos(ax + b)}{a} + c$$

$$\int \cos(ax + b) \, dx = \frac{\sin(ax + b)}{a} + c$$

$$\int \sec^2 ax \, dx = \frac{\tan(ax + b)}{a} + c$$

where $a (\neq 0)$, b and c are constants and angle x is in radians.

(c) **Exponential Functions**

$$\int e^x dx = e^x + c$$

$$\int e^{ax+b} dx = \frac{e^{ax+b}}{a} + c, a \neq 0$$

where a , b and c are constants.

3. Definite integrals

If $\int f(x)dx = h(x) + c$, where c is a constant, then

$$\int_a^b f(x) dx = [h(x)]_a^b = h(b) - h(a)$$

a is called the lower limit, b is called the upper limit and the whole integration is called the definite integration.

4. Some Results on Definite Integrals

$$(a) \int_b^b f(x) dx = 0$$

$$(b) \int_a^b f(x) dx = - \int_b^a f(x) dx$$

$$(c) \int_a^b f(x) dx + \int_b^c f(x) dx = \int_a^c f(x) dx$$

Worked Examples:

Example 1

Integrate each of the following with respect to x .

$$(a) \frac{x^4 + 7x}{x^3} \quad (b) \left(2x - \frac{1}{2}\right)^2 \quad (c) (3x + 2)^3$$

Solution:

$$\begin{aligned} (a) \int \frac{x^4 + 7x}{x^3} dx &= \int (x + 7x^{-2}) dx \\ &= \frac{x^2}{2} - \frac{7}{x} + c \end{aligned} \quad \text{where } c \text{ is a constant.}$$

$$\begin{aligned} (b) \int \left(2x - \frac{1}{2}\right)^2 dx &= \int \left(4x^2 - 2x + \frac{1}{4}\right) dx \\ &= \frac{4x^3}{3} - x^2 + \frac{x}{4} + c \end{aligned} \quad \text{where } c \text{ is constant.}$$

$$(c) \int (3x+2)^3 dx = \frac{(3x+2)^4}{3(4)} + c$$

$$= \frac{(3x+2)^4}{12} + c$$

where c is constant.

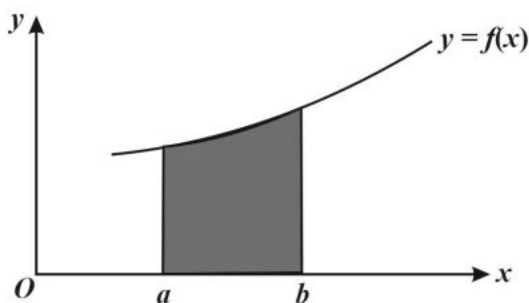
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TOPIC- 20

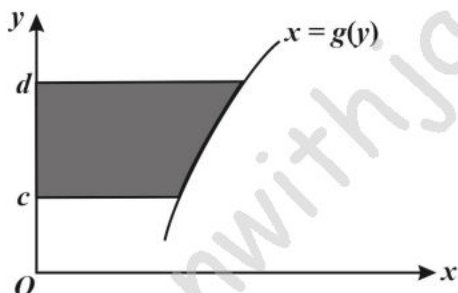
Area of a Region

Revision Notes

- The area bounded by the curve $y = f(x)$, the lines $x = a$, $x = b$ and the x -axis is given by $\int_a^b f(x) \, dx$.



- The area bounded by the curve $x = g(y)$, the lines $y = c$, $y = d$ and the y -axis is given by $\int_c^d g(y) \, dy$.

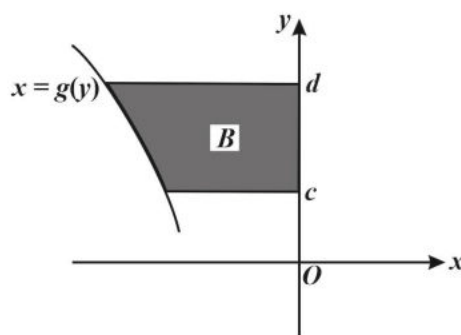
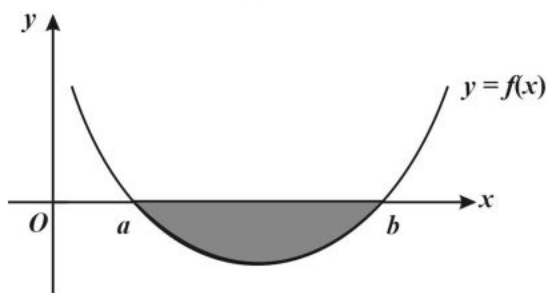


- Area is positive when it is above the x -axis or on the right of the y -axis. On the reverse, the area will be negative.

Therefore, Shaded area $A = - \int_a^b f(x) \, dx$ or $\left| \int_a^b f(x) \, dx \right|$

Shaded area $B = - \int_c^d g(y) \, dy$ or $\left| \int_c^d g(y) \, dy \right|$

A



O Level Additional Mathematicswhat they mean

Simultaneous Equations

	When the question ask	It means
1.	Find the value of h and k such that the two equations $3x + 2y = 4$ – Eqn (1) $6x + hy = k$ – Eqn (2) have no unique solutions. No unique solution also means the matrix $\begin{pmatrix} 3 & 2 \\ 6 & h \end{pmatrix}$ is singular.	For no unique solution, determinant of coefficient matrix $\begin{pmatrix} 3 & 2 \\ 6 & h \end{pmatrix}$ must be equal to zero. Hence, $h = 4$ No unique solutions can mean either: (a) Infinite Solutions (b) No solutions (a) For Infinite solutions Eqn (2) must be a multiple of Eqn (1). Which means these 2 equations produced the same line when plotted. Hence $k = 8$ (b) For No solution Eqn (2) is not a multiple of Eqn (1). Which means these 2 equations produced parallel lines when plotted. Hence k can take on any real value other than 8.

Quadratic Equations and Inequalities

	When the question says	It means
1.	the expression $ax^2 + bx + c$ is always positive	$a > 0$ and $b^2 - 4ac < 0$
2.	the expression $ax^2 + bx + c$ is never negative	$a > 0$ and $b^2 - 4ac \leq 0$
3.	the expression $ax^2 + bx + c$ is always negative	$a < 0$ and $b^2 - 4ac < 0$
4.	the expression $ax^2 + bx + c$ is never positive	$a < 0$ and $b^2 - 4ac \leq 0$
5.	the line $y + 2x = 8$ is a tangent to the curve $y = 2x^3 - 6x + k$,	$8 - 2x = 2x^2 - 6x + k$ $2x^2 - 4x + k - 8 = 0$ and $b^2 - 4ac = 0$

	When the question says	It means
6.	the line $y + 2x = 8$ meets the curve $y = 2x^2 - 6x + k$,	$8 - 2x = 2x^2 - 6x + k$ $2x^2 - 4x + k - 8 = 0$ and $b^2 - 4ac \geq 0$
7.	the line $y + 2x - 8$ does not meet the curve $y = 2x^2 - 6x + k$,	$8 - 2x = 2x^2 - 6x + k$ $2x^2 - 4x + k - 8 = 0$ and $b^2 - 4ac < 0$
8.	the line $y + 2x = 8$ intersect/cuts the curve $y = 2x^2 - 6x + k$ at two distinct points.	$8 - 2x = 2x^2 - 6x + k$ $2x^2 - 4x + k - 8 = 0$ and $b^2 - 4ac > 0$

Polynomials

	When the question says	It means
1.	find the remainder when $f(x)$ is divided by $(x - a)$	Calculate the value of $f(a)$
2.	that $f(x)$ is exactly divisible by $x^2 - 2x - 3$	$f(3) = 0$ and $f(-1) = 0$

Trigonometric Functions

	When the questions are	It means
1.	For the trigonometric functions : (i) $a \sin bx + c$ OR (ii) $a \cos bx + c$ (iii) $a \tan bx + c$	Amplitude of the function is a , Period of the function is $\frac{360^\circ}{b}$ or $\frac{2\pi}{b}$ Maximum value of the function is $a + c$. Minimum value of the function is $c - a$. For tangent function, period is $\frac{180^\circ}{b}$ or $\frac{\pi}{b}$. There is no amplitude or maximum or minimum value as tangent function can take on any real value.

Differentiation

	When the question says	It means
1.	show that $y = f(x)$ is an increasing function....	show that $\frac{dy}{dx} > 0$
2.	show that $y = f(x)$ is a decreasing function....	show that $\frac{dy}{dx} < 0$
3.	Find the range of values of x for which y is decreasing	Find the range of values of x for which $\frac{dy}{dx} < 0$
4.	Find the possible values of k for which the x -axis is a tangent to the curve $y = x^3 + 3x^2 - 9x + k$	<p>Step 1). Find the value of x such that $\frac{dy}{dx} = 0$</p> <p>Step 2). Substitute the value(s) of x in step 1 into the equation $x^3 + 3x^2 - 9x + k = 0$ to find k.</p>