

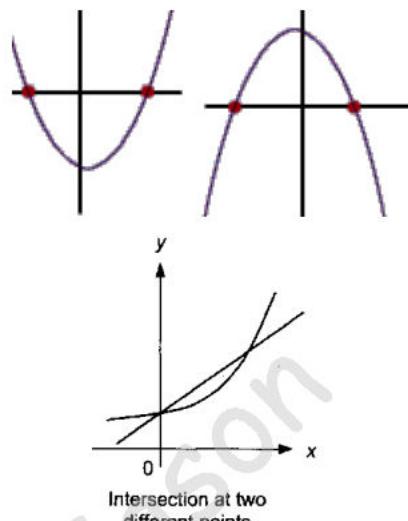
*Formulas highlighted in yellow are found in the formula list of the exam paper.

Quadratic Equation

$$b^2 - 4ac > 0$$

Real and Distinct Roots
/ Unequal Roots

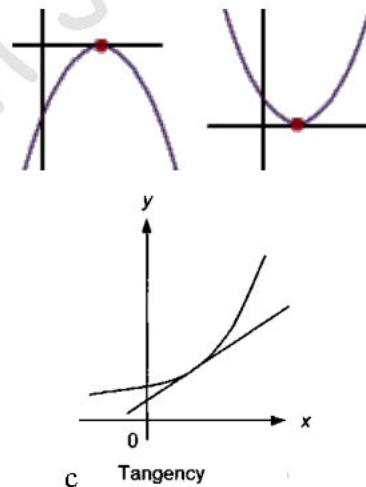
Curve and line intersect at 2 points



$$b^2 - 4ac = 0$$

Real and Equal Roots/Repeat Roots/
Coincident Roots.

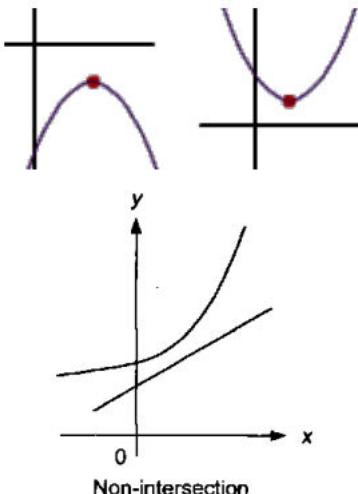
Curve and line intersect at 1 point.



$$b^2 - 4ac < 0$$

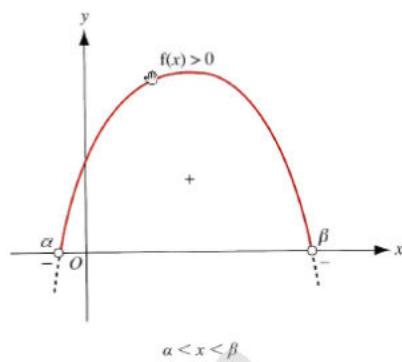
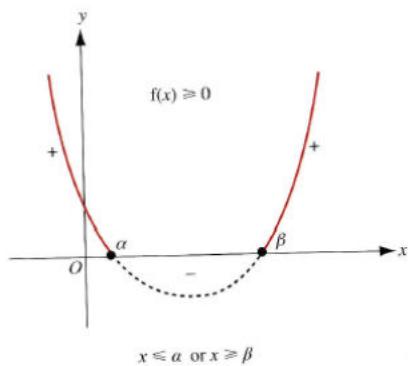
Imaginary roots
Also known as Complex Roots.

Curve and line intersect do not intersect.

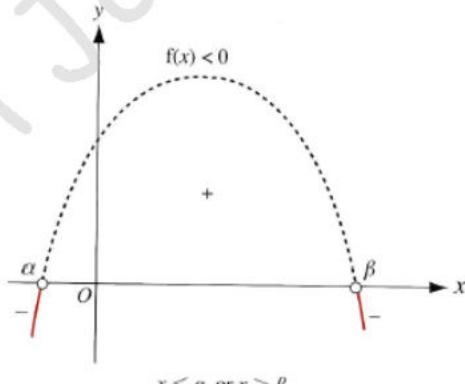
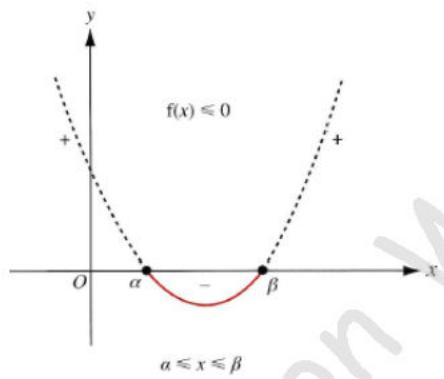


Quadratic Inequalities

When $y > 0$ (above the x-axis)



When $y < 0$ (below the x-axis)



Indices

Same base numbers

$$x^a \times x^b = x^{a+b}$$

Add the powers

Same power

$$a^m \times b^m = (a \times b)^m$$

Multiply the base numbers

$$x^0 = 1 \text{ if } x \neq 0$$

Note: $x^a + x^b \neq x^{a+b}$

$$x^a - x^b \neq x^{a-b}$$

Same base numbers

$$\frac{x^a}{x^b} = x^{a-b}$$

Subtract the powers

Same power

$$\frac{a^m}{b^m} = \left(\frac{a}{b}\right)^m$$

Divide the base numbers

$$(x^a)^b \neq x^a \times x^b$$

$$\text{NOTE: } (x^a)^b = x^{a \times b}$$

Other Laws of Indices

$$x^{-a} = \frac{1}{x^a}$$

$$x^{\frac{1}{b}} = \sqrt[b]{x^1}$$

$$x^{\frac{a}{b}} = \sqrt[b]{x^a}$$

$$x^{-\frac{1}{b}} = \frac{1}{x^{\frac{1}{b}}} = \frac{1}{\sqrt[b]{x^1}}$$

$$x^{-a} = x^a$$

$$\left(\frac{x}{y}\right)^{-a} = \left(\frac{y}{x}\right)^a$$

$$x^{-\frac{a}{b}} = \frac{1}{x^{\frac{a}{b}}} = \frac{1}{\sqrt[b]{x^a}}$$

NOTE: You can only use the laws of indices if either the base number or the power is the same.

Surds

$$\sqrt{a} \times \sqrt{b} = \sqrt{a \times b}$$

$$\sqrt{a} \div \sqrt{b} = \sqrt{\frac{a}{b}}$$

$$\sqrt{a} \times \sqrt{a} = (\sqrt{a})^2 = a$$

$$m\sqrt{a} \times n\sqrt{b} = m \times n \sqrt{a \times b}$$

$$m\sqrt{a} + n\sqrt{a} = (m+n)\sqrt{a}$$

$$m\sqrt{a} - n\sqrt{a} = (m-n)\sqrt{a}$$

Rationalizing Denominator

$$\frac{a}{\sqrt{x}-\sqrt{y}} \times \frac{\sqrt{x}+\sqrt{y}}{\sqrt{x}+\sqrt{y}} = \frac{a(\sqrt{x}+\sqrt{y})}{(\sqrt{x})^2 - (\sqrt{y})^2} = \frac{a\sqrt{x}+a\sqrt{y}}{x-y}$$

$$\frac{a}{\sqrt{x}+\sqrt{y}} \times \frac{\sqrt{x}-\sqrt{y}}{\sqrt{x}-\sqrt{y}} = \frac{a(\sqrt{x}-\sqrt{y})}{(\sqrt{x})^2 - (\sqrt{y})^2} = \frac{a\sqrt{x}-a\sqrt{y}}{x-y}$$

Note: $\sqrt{a+b} \neq \sqrt{a} + \sqrt{b}$ $\sqrt{a-b} \neq \sqrt{a} - \sqrt{b}$

Polynomials & Partial Fractions

$$x^3 + y^3 = (x+y)(x^2 - xy + y^2)$$

$$x^3 - y^3 = (x-y)(x^2 + xy + y^2)$$

Linear Factor

$$\frac{mx+n}{(ax+b)(cx-d)} = \frac{A}{(ax+b)} + \frac{B}{(cx+d)}$$

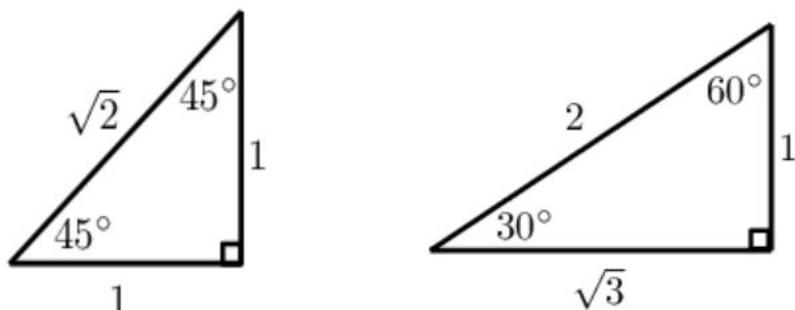
Repeat Factors

$$\frac{mx+n}{(ax+b)(cx-d)^2} = \frac{A}{(ax+b)} + \frac{B}{(cx+d)} + \frac{C}{(cx+d)^2}$$

Quadratic Factors

$$\frac{mx+n}{(ax+b)(cx^2-d)} = \frac{A}{(ax+b)} + \frac{Bx+C}{(cx^2+d)}$$

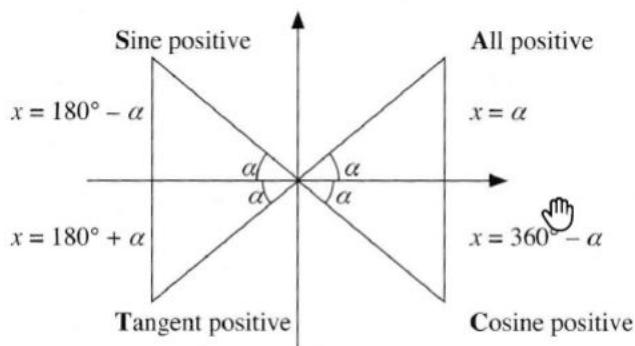
Note: If the highest coefficient of the NUMERATOR is the SAME or LARGER than the DENOMINATOR. Do **LONG DIVISION** before partial fractions.

Trigonometry

	30°	45°	60°
\sin	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{\sqrt{3}}{2}$
\cos	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$
\tan	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$

Principal Values of $\sin^{-1} x$, $\cos^{-1} x$, $\tan^{-1} x$,

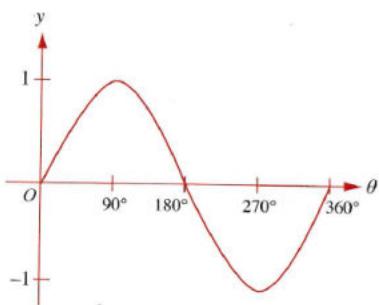
$$-\frac{\pi}{2} \leq \sin^{-1} x \leq \frac{\pi}{2} \quad 0 \leq \cos^{-1} x \leq \pi \quad -\frac{\pi}{2} < \tan^{-1} x < \frac{\pi}{2}$$



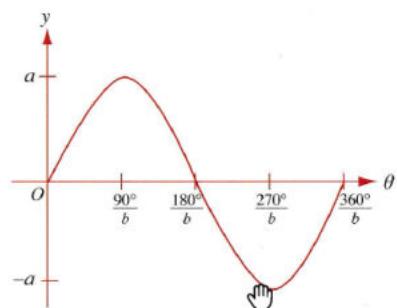
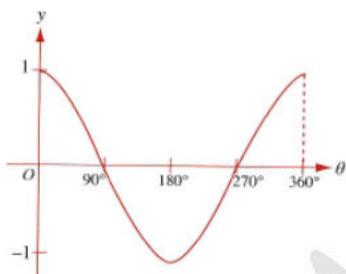
"Add Sugar To Coffee"

or

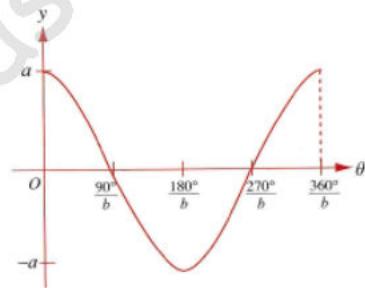
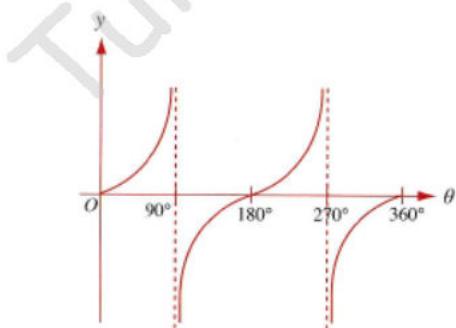
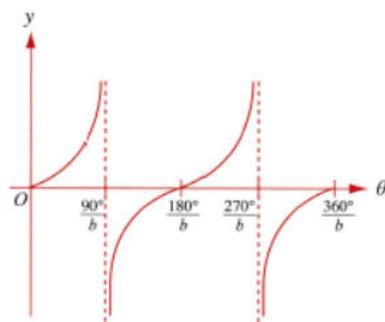
"All Science/Social Studies Teachers Crazy"

1. (a) $y = \sin \theta$ Period = 360° or 2π

Amplitude = 1

(b) $y = a \sin b\theta$ Period = $\frac{360^\circ}{b}$ or $\frac{2\pi}{b}$ Amplitude = a 2. (a) $y = \cos \theta$ Period = 360° or 2π

Amplitude = 1

(b) $y = a \cos b\theta$ Period = $\frac{360^\circ}{b}$ or $\frac{2\pi}{b}$ Amplitude = a 3. (a) $y = \tan \theta$ Period = 180° or π (b) $y = a \tan b\theta$ Period = $\frac{180^\circ}{b}$ or $\frac{\pi}{b}$

$$\cos(-\theta) = \cos \theta$$

$$\tan \theta = \frac{\sin \theta}{\cos \theta}$$

$$\sin(-\theta) = -\sin \theta$$

$$\tan(-\theta) = -\tan \theta$$

$$\sec \theta = \frac{1}{\cos \theta}$$

$$\cos(90^\circ \pm \theta) = \mp \sin \theta$$

$$\cosec \theta = \frac{1}{\sin \theta}$$

$$\sin(90^\circ \pm \theta) = \cos \theta$$

$$\cot \theta = \frac{1}{\tan \theta} = \frac{\cos \theta}{\sin \theta}$$

$$\tan(90^\circ \pm \theta) = \mp \tan \theta$$

$$\sin^2 \theta + \cos^2 \theta = 1 \quad 1 + \cot^2 \theta = \cosec^2 \theta \quad 1 + \tan^2 \theta = \sec^2 \theta$$

Compound Angle Formula

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$

Double Angle Formula

$$\sin 2A = 2 \sin A \cos A$$

$$\cos 2A = 2 \cos^2 A - 1 / \cos 2A = \cos^2 A - \sin^2 A / \cos 2A = 1 - 2 \sin^2 A$$

$$\tan 2A = \frac{2 \tan A}{1 - \tan^2 A}$$

Half Angle Formula

$$\sin \frac{A}{2} = \pm \sqrt{\frac{1 - \cos A}{2}}$$

$$\cos \frac{A}{2} = \pm \sqrt{\frac{1 + \cos A}{2}}$$

$$\tan \frac{A}{2} = \sqrt{\frac{1 - \cos A}{1 + \cos A}} = \frac{1 - \cos A}{\sin A}$$

R-Formula

$$a \cos \theta \pm b \sin \theta = R \cos(\theta \mp \alpha)$$

Where $R = \sqrt{a^2 + b^2}$

$$a \sin \theta \pm b \cos \theta = R \sin(\theta \pm \alpha)$$

$$\tan \alpha = \frac{b}{a}$$

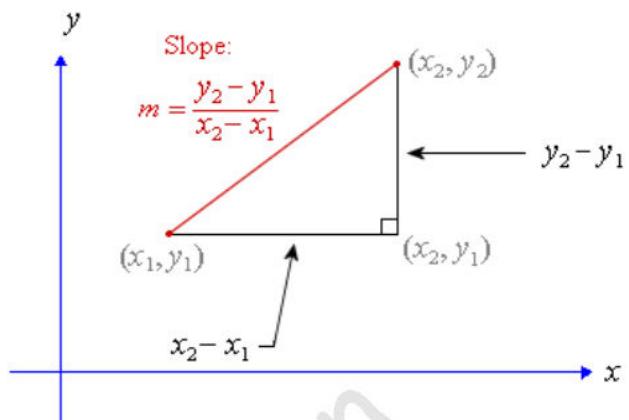
Coordinate Geometry

$$\text{Gradient}(m) = \frac{y_2 - y_1}{x_2 - x_1}$$

General Equation $Y - y_1 = m(X - x_1)$

where (x_1, y_1) and (x_2, y_2) are point on the graph.

OR you may use $Y = mX + c$

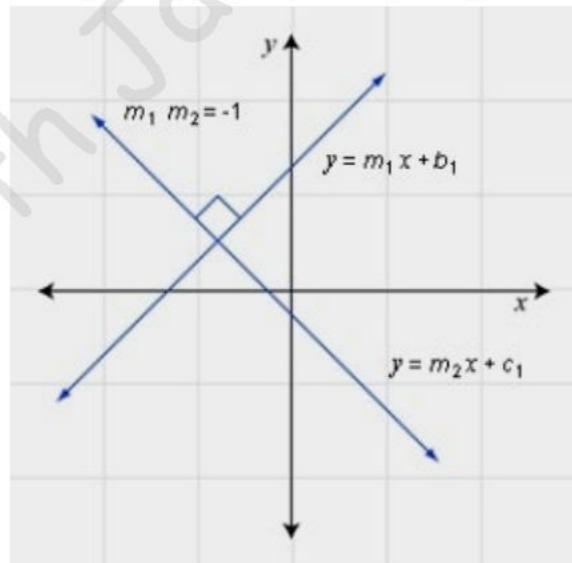


Mid-point of a line

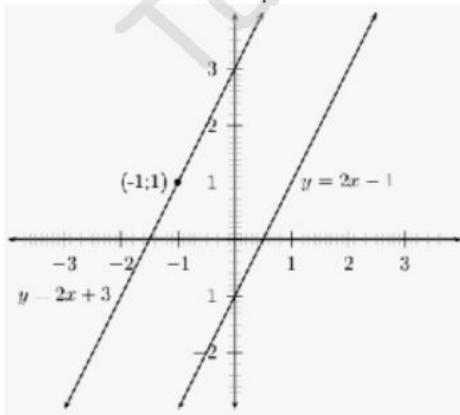
$$(x_m = \frac{x_1 + x_2}{2}, y_m = \frac{y_1 + y_2}{2})$$

Distance between two points

$$\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$



When two lines are parallel



They have the same gradient (m)

If two lines have perpendicular gradient i.e. 90° to each other.

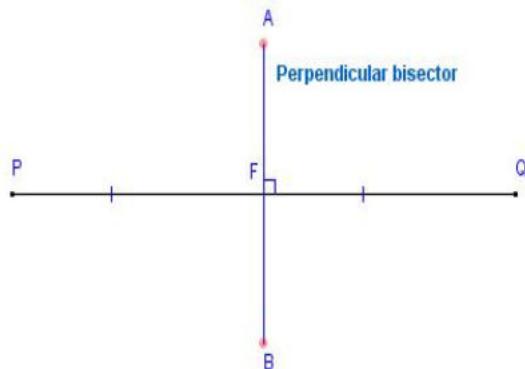
$$m_1 = -\frac{1}{m_2} \quad \text{or} \quad m_1 \times m_2 = -1$$

Perpendicular Bisector of line PQ $y = m_2x + c$

1. The perpendicular lines AB intersects PQ at 90°

$$m_2(AB) = -\frac{1}{m_1(PQ)}$$

or $m_1 \times m_2 = -1$



2. The perpendicular line AB cuts the mid-point of the line PQ.

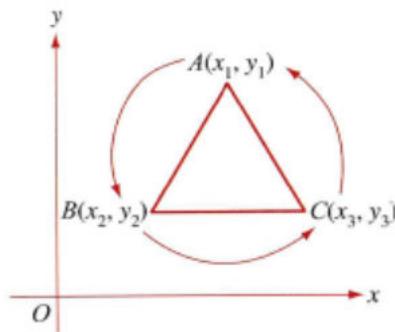
$$\text{Mid-point } PQ = \left(x_m = \frac{x_2 + x_1}{2}, y_m = \frac{y_2 + y_1}{2} \right)$$

3. To find the y-intercept (c)

Substitute $x_m \rightarrow x$ and $y_m \rightarrow y$ into $y = m_2x + c$

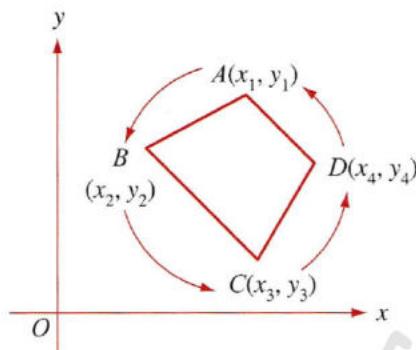
Area of Plane Figure (Polygon Figure)

$$\begin{aligned} \text{Area of } \triangle ABC &= \frac{1}{2} \left| \begin{matrix} x_1 & x_2 & x_3 & x_1 \\ y_1 & y_2 & y_3 & y_1 \end{matrix} \right| \\ &= \frac{1}{2} (x_1 y_2 + x_2 y_3 + x_3 y_1 - x_1 y_3 - x_3 y_2 - x_2 y_1) \end{aligned}$$



$$\text{Area of } ABCD = \frac{1}{2} \left| \begin{matrix} x_1 & x_2 & x_3 & x_4 & x_1 \\ y_1 & y_2 & y_3 & y_4 & y_1 \end{matrix} \right|$$

↙ = $\frac{1}{2} (x_1 y_2 + x_2 y_3 + x_3 y_4 + x_4 y_1 - x_1 y_4 - x_4 y_3 - x_3 y_2 - x_2 y_1)$



Note: Remember to repeat the first set of coordinates (x_1, y_1) in the formula.

It does not matter if you insert the coordinates clockwise or anti-clockwise into the formula. The **|modulus|** sign will convert any negative value positive while a positive value remains positive.

Circles

Equation of a Circle

(Standard Form)

$$(x - a)^2 + (y - b)^2 = r^2$$

$$r = \sqrt{(x - a)^2 + (y - b)^2}$$

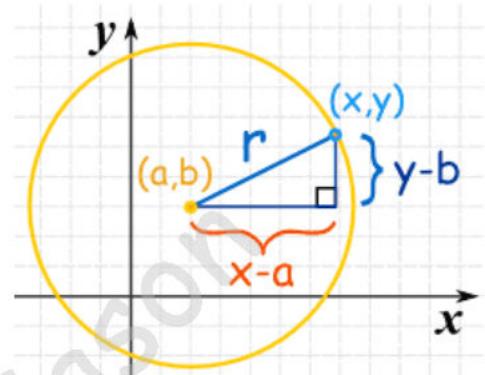
Where (a,b) are the coordinates of the centre of circle
and r is the radius

(General Form)

$$x^2 + y^2 + 2gx + 2fy + c = 0$$

where $a = -g$, $b = -f$

$$r = \sqrt{g^2 + f^2 - c}$$



Differentiation

$$\frac{dy}{dx}(ax^n) = anx^{n-1}$$

Where 'a' and 'n'
are constants. .

Differentiate Constant

$$\frac{dy}{dx}(a) = 0$$

Sum / Difference of Function

$$\frac{d}{dx}(u \pm v) = \frac{du}{dx} \pm \frac{dv}{dx}$$

Chain Rule

$$\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx}$$

Quotient Rule

$$\frac{d}{dx}\left(\frac{u}{v}\right) = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$$

Product Rule

$$\frac{d}{dx}(uv) = u\frac{dv}{dx} + v\frac{du}{dx}$$

Differentiation of Trigonometry

$$\frac{d}{dx}(\sin x) = \cos x$$

$$\frac{d}{dx}(\tan x) = \sec^2 x$$

$$\frac{d}{dx}(\cos x) = -\sin x$$

Use Chain Rule to differentiate the functions below.

$$\frac{d}{dx}[a \sin(bx + c)] = a \times \cos(bx + c) \times b$$

$$\frac{d}{dx}[a \cos(bx + c)] = a \times -\sin(bx + c) \times b$$

$$\frac{d}{dx}[a \tan(bx + c)] = a \times \sec^2(bx + c) \times b$$

$$\frac{d}{dx} [a \sin^n (bx + c)] = a \times n \times \sin^{n-1}(bx + c) \times \cos(bx + c) \times b$$

$$\frac{d}{dx} [a \cos^n (bx + c)] = a \times n \times \cos^{n-1}(bx + c) \times -\sin(bx + c) \times b$$

$$\frac{d}{dx} [a \tan^n (bx + c)] = a \times n \times \tan^{n-1}(bx + c) \times \sec^2(bx + c) \times b$$

Exponential/Natural Logarithm Function

$$\frac{d}{dx} (e^{ax+b}) = ae^{ax+b}$$

'a' is a constant

$$\frac{d}{dx} (e^x) = e^x$$

$$\frac{d}{dx} (\ln x) = \frac{1}{x} \text{ (where } x > 0)$$

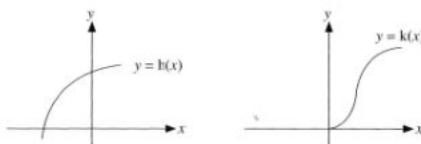
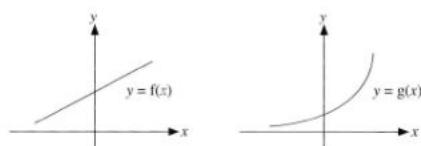
$$\frac{d}{dx} [\ln(ax + b)] = \frac{a}{ax + b}$$

(where $ax + b > 0$)

Increasing Function

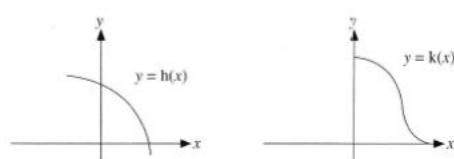
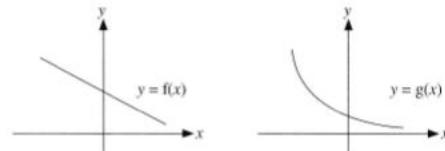
If y is an increasing function (y increases as x increases), the gradient is positive, i.e. $\frac{dy}{dx} > 0$.

e.g.

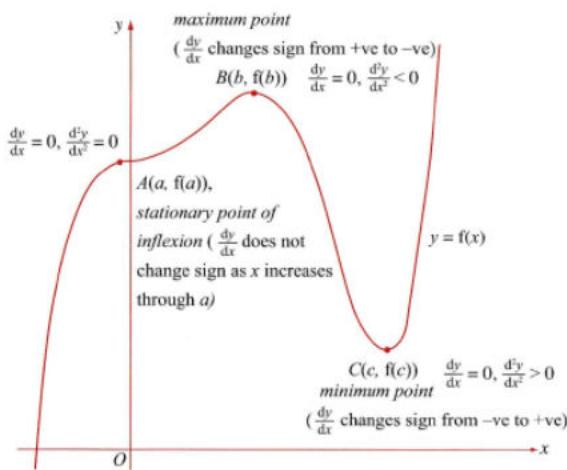


Decreasing Function

If y is a decreasing function (y decreases as x increases), the gradient is negative, i.e. $\frac{dy}{dx} < 0$.



Turning & Inflection Points



Note:

When $\frac{d^2y}{dx^2} = 0$ it does not necessarily

mean that it is a point of inflection.

There are some graphs that will show

$\frac{d^2y}{dx^2} = 0$ even though they are turning

points (maximum or minimum point). You will then need to carry out a 1st order derivative test.

1st order derivative test

Maximum point

	x^-	x_0	x^+
$\frac{dy}{dx}$	> 0	0	< 0
slope	/	-	\
stationary point			

Minimum point

	x^-	x_0	x^+
$\frac{dy}{dx}$	< 0	0	> 0
slope	\	-	/
stationary point			

Point of inflection

	x^-	x_0	x^+
$\frac{dy}{dx}$	> 0	0	> 0
slope	/	-	/
stationary point			

Point of inflection

	x^-	x_0	x^+
$\frac{dy}{dx}$	< 0	0	< 0
slope	\	-	\
stationary point			

Integration

$$\int ax^n dx = \frac{ax^{n+1}}{n+1} + c$$

where $n \neq -1$

Integrate
Constant
 $\int a dx = ax + c$

Product rule $n \neq 1, a \neq 0$; a & b are constants

$$\int (ax+b)^n dx = \frac{(ax+b)^{n+1}}{(n+1)a} + c$$

Product of constant and a function

$$\int cf(x)dx = a \int f(x)dx$$

Sum and Difference of function

$$\int [\alpha f(x) \pm \beta g(x)] dx = \alpha \int f(x)dx \pm \beta \int g(x)dx$$

Integration of Trigonometry

$$\int \cos x dx = \sin x + c$$

$$\int \sin x dx = -\cos x + c$$

$$\int \sec^2 x dx = \tan x + c$$

$$\int a \cos bx dx = \frac{a \sin bx}{b} + c$$

$$\int a \sin bx dx = \frac{-a \cos bx}{b} + c$$

$$\int a \sec^2 bx dx = \frac{a \tan bx}{b} + c$$

Integration of exponential and Ln

$$\int \frac{1}{x} dx = \ln x + c \text{ where } x > 0$$

$$\int \frac{1}{ax+b} dx = \frac{\ln(ax+b)}{a} + C$$

$$\int \frac{1}{ax^n + b} dx = \frac{\ln(ax^n + b)}{a \times n \times x^{n-1}} + C$$

$$\int e^x dx = e^x + c$$

$$\int e^{ax+b} dx = \frac{e^{ax+b}}{a} + C$$

$$\int e^{ax^n+b} dx = \frac{e^{ax^n+b}}{a \times n \times x^{n-1}} + C$$

Basic Properties of Definite Integral

$$\int_a^a f(x) dx = 0$$

$$\int_a^b f(x) dx = - \int_b^a f(x) dx$$

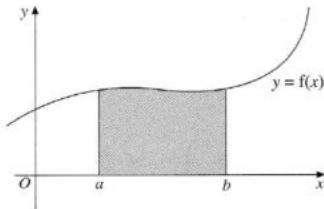
$$\int_a^b f(x) dx = \int_c^b f(x) dx + \int_a^c f(x) dx$$

Integration of Area

For a region above the x-axis:

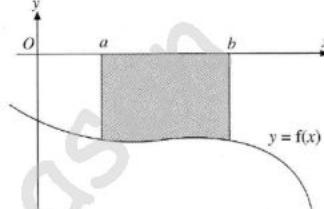
Area bounded by the curve $y = f(x)$, the lines $x = a$ and $x = b$ and the x-axis is

$$\int_a^b f(x) dx.$$


For a region below the x-axis:

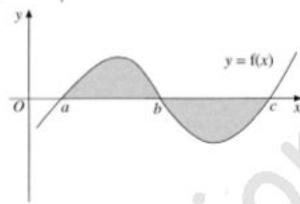
Area bounded by the curve $y = f(x)$, the lines $x = a$ and $x = b$ and the x-axis is

$$\left| \int_a^b f(x) dx \right|.$$


For an area enclosed above and below the x-axis:

Area bounded by the curve $y = f(x)$ and the x-axis as shown below is

$$\int_a^b f(x) dx + \left| \int_b^c f(x) dx \right|$$



Note:

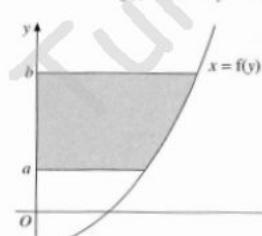
The **|modulus sign|** convert a negative value to positive, and if the value is positive, it remains positive.

Area cannot have a negative value.

For a region on the right side of the y-axis:

Area bounded by the curve $x = f(y)$, the lines $y = a$ and $y = b$ and the y-axis is

$$\int_a^b f(y) dy.$$


For a region on the left side of the y-axis:

Area bounded by the curve $x = f(y)$, the lines $y = a$ and $y = b$ and the y-axis is

$$\left| \int_a^b f(y) dy \right|.$$

