## Tuition

A-Math Formula List - Additional Math (4049)
*Formulas highlighted in yellow are found in the formula list of the exam paper.

| Quadratic Equation |  |
| :---: | :---: |
| $b^{2}-4 a c>0$ <br> Real and Distinct Roots <br> / Unequal Roots <br> Curve and line intersect at 2 points |   |
| $b^{2}-4 a c=0$ <br> Real and Equal <br> Roots/Repeat Roots/ <br> Coincident Roots. <br> Curve and line intersect at 1 point. |   |
| $b^{2}-4 a c<0$ <br> Imaginary roots <br> Also known as Complex <br> Roots. <br> Curve and line intersect do not intersect. |   |

## Tuition

## A-Math Formula List - Additional Math (4049)

## Quadratic Inequalities



## Tuition

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Other Laws of Indices

| $x^{-a}=\frac{1}{x^{a}}$ | $x^{\frac{1}{b}}=\sqrt[b]{x^{1}}$ | $x^{\frac{a}{b}}=\sqrt[b]{x^{a}}$ |
| :--- | :--- | :--- |
| $x^{-\frac{1}{b}}=\frac{1}{\frac{1}{b}}=\frac{1}{\sqrt[b]{x^{1}}}$ | $\frac{1}{x^{-a}}=x^{a}$ | $\left(\frac{x}{y}\right)^{-a}=\left(\frac{y}{x}\right)^{a}$ |
| $x^{\frac{a}{b}}$ |  |  |
| $x^{-\frac{a}{b}}=\frac{1}{\frac{a}{b}}=\frac{1}{\sqrt[b]{x^{a}}}$ | Note: You can only use the laws of indices if either the base <br> number or the power is the same. |  |

## Surds

| $\sqrt{a} \times \sqrt{b}=\sqrt{a \times b}$ | $\sqrt{a} \div \sqrt{b}=\sqrt{\frac{a}{b}}$ |
| :--- | :--- |
| $\sqrt{a} \times \sqrt{a}=(\sqrt{a})^{2}=a$ | $m \sqrt{a} \times n \sqrt{b}=m \times n \sqrt{a \times b}$ |
| $m \sqrt{a}+n \sqrt{a}=(m+n) \sqrt{a}$ | $m \sqrt{a}-n \sqrt{a}=(m-n) \sqrt{a}$ |

## Rationalizing Denominator

$$
\frac{a}{\sqrt{x}-\sqrt{y}} \times \frac{\sqrt{x}+\sqrt{y}}{\sqrt{x}+\sqrt{y}}=\frac{a(\sqrt{x}+\sqrt{y})}{(\sqrt{x})^{2}-(\sqrt{y})^{2}}=\frac{a \sqrt{x}+a \sqrt{y}}{x-y}
$$

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$\frac{a}{\sqrt{x}+\sqrt{y}} \times \frac{\sqrt{x}-\sqrt{y}}{\sqrt{x}-\sqrt{y}}=\frac{a(\sqrt{x}-\sqrt{y})}{(\sqrt{x})^{2}-(\sqrt{y})^{2}}=\frac{a \sqrt{x}-a \sqrt{y}}{x-y}$

Note: $\quad \sqrt{a+b} \neq \sqrt{a}+\sqrt{b} \quad \sqrt{a-b} \neq \sqrt{a}-\sqrt{b}$

## Polynomials \& Partial Factions

$$
x^{3}+y^{3}=(x+y)\left(x^{2}-x y+y^{2}\right) \quad x^{3}-y^{3}=(x-y)\left(x^{2}+x y+y^{2}\right)
$$

## Linear Factor

$$
\frac{m x+n}{(a x+b)(c x-d)}=\frac{A}{(a x+b)}+\frac{B}{(c x+d)}
$$

## Repeat Factors

$$
\frac{m x+n}{(a x+b)(c x-d)^{2}}=\frac{A}{(a x+b)}+\frac{B}{(c x+d)}+\frac{C}{(c x+d)^{2}}
$$

## Quadratic Factors

$\frac{m x+n}{(a x+b)\left(c x^{2}-d\right)}=\frac{A}{(a x+b)}+\frac{B x+C}{\left(c x^{2}+d\right)}$

Note: If the highest coefficient of the NUMERATOR is the SAME or LARGER that the DENOMINATOR. Do LONG DIVI SION before partial fractions.

## Logarithms

| $\log _{a} 1=0$ | $\log _{a} a=1$ |
| :--- | :--- |
| Since $\lg =\log _{10}, \therefore \lg 10=1$ | Since $\ln =\log _{\mathrm{e},}, \therefore \ln _{\mathrm{e}} \mathrm{e}=1$ |
| $\log _{\mathrm{b}} \mathrm{m}^{\mathrm{a}}=\mathrm{a} \times \log _{\mathrm{b}} \mathrm{m}$ | $\mathrm{e}^{\ln \mathrm{a}}=\mathrm{a}$ |
| $\log _{\mathrm{b}} \mathrm{m}+\log _{\mathrm{b}} \mathrm{n}=\log _{\mathrm{b}}(\mathrm{m} \times \mathrm{n})$ |  |

Note: $\log _{\mathrm{b}} \mathrm{m}+\log _{\mathrm{b}} \mathrm{n} \neq \log _{\mathrm{b}} \mathrm{m} \times \log _{\mathrm{b}} \mathrm{n}$
$\log _{\mathrm{b}} \mathrm{m}-\log _{\mathrm{b}} \mathrm{n}=\log _{\mathrm{b}}\left(\frac{m}{n}\right)$

Note: $\log _{\mathrm{b}} \mathrm{m}-\log _{\mathrm{b}} \mathrm{n} \neq \frac{\log _{b} m}{\log _{\mathrm{b}} n}$

Change of Base

$$
\log _{v} u=\frac{\log _{a} u}{\log _{a} v} \quad \log _{v} u=\frac{\log _{u} u}{\log _{u} v}=\frac{1}{\log _{u} v}
$$

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If $\log _{a} \mathrm{~b}=\mathrm{x}, \therefore \mathrm{b}=\mathrm{a}^{\mathrm{x}}$

If $\log _{a} \mathrm{~b}=\log _{a} \mathrm{X}, \therefore \mathrm{b}=\mathrm{x}$

## Binomial Theorem

$$
\binom{n}{r}=\frac{n!}{r!(n-r)!} \quad\binom{n}{r}={ }^{n} C_{r}
$$

## General Term

$$
(a+b)^{n} \rightarrow\binom{n}{r}(a)^{n-r}(b)^{r}
$$

Remember: $\left(\frac{n}{0}\right)=1,\left(\frac{n}{1}\right)=n,\left(\frac{n}{2}\right)=\frac{n(n-1)}{1 \times 2},\left(\frac{n}{3}\right)=\frac{n(n-1)(n-2)}{1 \times 2 \times 3}$

## Since:

$$
(a+b x)^{n}=a^{n}+\binom{n}{1} a^{n-1}(b x)^{1}+\binom{n}{2} a^{n-2}(b x)^{2}+\binom{n}{3} a^{n-3}(b x)^{3} \ldots .\binom{n}{r} a^{n-r}(b x)^{r}+(b x)^{n}
$$

Therefore:

$$
(a+b x)^{n}=a^{n}+\frac{n}{1} a^{n-1}(b x)^{1}+\frac{(n)(n-1)}{1 \times 2} a^{n-2}(b x)^{2}+\frac{(n)(n-1)(n-2)}{1 \times 2 \times 3} a^{n-3}(b x)^{3} \ldots .+(b x)^{n}
$$

## Alternate formula if $a=1$

$$
(1+k x)^{n}=1+\binom{n}{1}(k x)^{1}+\binom{n}{2}(k x)^{2}+\ldots .\binom{n}{n-1}(k x)^{n-1}+(k x)^{n}
$$

## Tuition

A-Math Formula List - Additional Math (4049)

Trigonometry


|  | $30^{\circ}$ | $45^{\circ}$ | $60^{\circ}$ |
| :---: | :---: | :---: | :---: |
| $\sin$ | $\frac{1}{2}$ | $\frac{1}{\sqrt{2}}$ | $\frac{\sqrt{3}}{2}$ |
| $\cos$ | $\frac{\sqrt{3}}{2}$ | $\frac{1}{\sqrt{2}}$ | $\frac{1}{2}$ |
| $\tan$ | $\frac{1}{\sqrt{3}}$ | 1 | $\sqrt{3}$ |

Principal Values of $\sin ^{-1} x, \cos ^{-1} x, \tan ^{-1} x$,
$-\frac{\pi}{2} \leq \sin ^{-1} x \leq \frac{\pi}{2}$
$0 \leq \cos ^{-1} x \leq \pi$
$-\frac{\pi}{2}<\tan ^{-1} x<\frac{\pi}{2}$

"Add Sugar To Coffee"
or
"All Science/Social Studies Teachers Crazy"

## Tuition

## withor

1. (a) $y=\sin \theta$


I'eriod $=360^{\circ}$ or $2 \pi$
Amplitude - 1
2. (a) $y=\cos \theta$


Period $=360^{\circ}$ or $2 \pi$
Amplitude $=1$
(b) $y=a \sin b \theta$


Period $=\frac{360^{\circ}}{b}$ or $\frac{2 \pi}{b}$
Amplitude - $a$
(b) $y=a \cos b \theta$


Period $=\frac{360^{\circ}}{b}$ or $\frac{2 \pi}{b}$
Amplitude $=a$
3. (a) $y=\tan \theta$
(b) $y=a \tan b \theta$


Period $=180^{\circ}$ or $\pi$

$$
\text { Period }=\frac{180^{\circ}}{b} \text { or } \frac{\pi}{b}
$$

## Tuition

A-Math Formula List - Additional Math (4049)


## Tuition

A-Math Formula List - Additional Math (4049)

## Double Angle Formula

$$
\sin 2 A=2 \sin A \cos A
$$

$$
\cos 2 A=2 \cos ^{2} A-1 / \cos 2 A=\cos ^{2} A-\sin ^{2} A / \cos 2 A=1-2 \sin ^{2} A
$$

$$
\tan 2 A=\frac{2 \tan A}{1-\tan ^{2} A}
$$

Half Angle Formula

$$
\begin{aligned}
& \sin \frac{A}{2}= \pm \sqrt{\frac{1-\cos A}{2}} \\
& \cos \frac{A}{2}= \pm \sqrt{\frac{1+\cos A}{2}} \\
& \tan \frac{A}{2}=\sqrt{\frac{1-\cos A}{1+\cos A}}=\frac{1-\cos A}{\sin A}
\end{aligned}
$$

## R-Formula

| $a \operatorname{Cos} \theta \pm b \operatorname{Sin} \theta=R \operatorname{Cos}(\theta \mp \alpha)$ | Where $R=\sqrt{a^{2}+b^{2}}$ |
| :--- | :--- |
| $\sin \theta \pm b \operatorname{Cos} \theta=R \operatorname{Sin}(\theta \pm \alpha)$ |  |

## Tuition

A-Math Formula List - Additional Math (4049)

## Coordinate Geometry

$\operatorname{Gradient}(m)=\frac{y_{2}-y_{1}}{x_{2}-x_{1}}$
General Equation $Y-y_{1}=m\left(X-x_{1}\right)$
where $\left(x_{1}, y_{1}\right)$ and $\left(x_{2}, y_{2}\right)$ are
point on the graph.
OR you may use $Y=m X+c$

Mid-point of a line

$$
\left(x_{m}=\frac{x_{1}+x_{2}}{2}, y_{m}=\frac{y_{1}+y_{2}}{2}\right)
$$

Distance between two points

$$
\sqrt{\left(x_{2}-x_{1}\right)^{2}+\left(y_{2}-y_{1}\right)^{2}}
$$

When two lines are parallel


They have the same gradient (m)

## Tuition

Perpendicular Bisector of line $P Q y=m_{2} x+c$

1. The perpendicular lines $A B$ intersects $P Q$ at $90^{\circ}$
$m_{2}(A B)=-\frac{1}{m_{1}(P Q)}$
or $m_{1} \times m_{2}=-1$
2. The perpendicular line $A B$ cuts the mid-point of the line $P Q$.

Mid-point $\mathrm{PQ}=\left(x_{m}=\frac{x_{2}+x_{1}}{2}, y_{m}=\frac{y_{2}+y_{1}}{2}\right)$
3. To find the $y$-intercept (c)

Substitute $x m \rightarrow x$ and $y m \rightarrow y$ into $y=m_{2} x+c$

Area of Plane Figure (Polygon Figure)

$$
\begin{aligned}
& \text { 1. Area of } \triangle A B C=\frac{1}{2}\left|\begin{array}{lll}
x_{1} & x_{2} & x_{3} \\
y_{1}<y_{2}<y_{3} & x_{1} \\
y_{3}>y_{1}
\end{array}\right| \\
& =\frac{1}{2}\left(x_{1} y_{2}+x_{2} y_{3}+x_{3} y_{1}-x_{1} y_{3}-x_{3} y_{2}-x_{2} y_{1}\right)
\end{aligned}
$$



## Tuition

## A-Math Formula List - Additional Math (4049)

$$
\begin{aligned}
\text { Area of } A B C D & =\frac{1}{2}\left|\begin{array}{l}
x_{1}, x_{2} \\
y_{1}, y_{2}, x_{3}, y_{3}, x_{4}, y_{4}, y_{1}
\end{array}\right| \\
\text { and } & =\frac{1}{2}\left(x_{1} y_{2}+x_{2} y_{3}+x_{3} y_{4}+x_{4} y_{1}-x_{1} y_{4}-x_{4} y_{3}-x_{3} y_{2}-x_{2} y_{1}\right)
\end{aligned}
$$



Note: Remember to repeat the first set of coordinates $\left(x_{1}, y_{1}\right)$ in the formula.
It does not matter if you insert the coordinates clockwise or anti-clockwise into the formula. The |modulus| sign will convert any negative value positive while a positive value remains positive.

## Tuition

## A-Math Formula List - Additional Math (4049)

## Circles

## Equation of a Circle

(Standard Form)

$$
\begin{aligned}
& (x-a)^{2}+(y-b)^{2}=r^{2} \\
& r=\sqrt{(x-a)^{2}+(y-b)^{2}}
\end{aligned}
$$

Where $(a, b)$ are the coordinates of the centre of circle and $r$ is the radius

(General Form)

$$
x^{2}+y^{2}+2 g x+2 f y+c=0
$$

where $a=-g, b=-f$

$$
r=\sqrt{g^{2}+f^{2}-c}
$$

## Tuition

A-Math Formula List - Additional Math (4049)

Proofs in Plane Geometry
Isosceles Triangle

## Tuition

A-Math Formula List - Additional Math (4049)
Equal Chord, Equal Distance from
Centre
Tangents from External Point
$\mathrm{BC}=\mathrm{BA}$
Congruent Triangles
If $A B=P Q, B C=Q R$ and $C A=R P$, then $\triangle A B C$ is congruent to $\triangle P Q R$

## Tuition

## withor A-Math Formula List - Additional Math (4049)

If $A B=P Q, A C=P R$ and $B \hat{A C}=Q \hat{P} R$, then $\triangle A B C$ is congruent to $\triangle P Q R$
(SAS Congruence Test).


If $A \hat{B C}=P \hat{Q} R, A \widehat{C} B=P \hat{R} Q$ and $B C=Q R$, then $\triangle A B C$ is congruent to $\triangle P Q R$ (ASA Congruence Test).


If $B \hat{A} C=Q \hat{P} R, A \hat{B} C=P \hat{Q} R$ and $B C=Q R$, then $\triangle A B C$ is congruent to $\triangle P Q R$ (AAS Congruence Test).


If $A C=X Z, A B=X Y$ or $B C=Y Z$, and $A \hat{B} C=X \hat{Y} Z=90^{\circ}$, then $\triangle A B C$ is congruent to $\triangle X Y Z$ (RHS Congruence Test).


## Tuition

## withor A-Math Formula List - Additional Math (4049)

Similar Triangles
If $B \hat{A} C=Q \hat{P} R$ and $A \hat{B} C=P \hat{Q} R$, then $\triangle A B C$ is similar to $\triangle P Q R$ (AA Similarity Test).


If $\frac{P Q}{A B}=\frac{Q R}{B C}=\frac{R P}{C A}$, then $\triangle A B C$ is similar to $\triangle P Q R$ (SSS Similarity Test).


If $\frac{P Q}{A B}=\frac{Q R}{B C}$ and $A \hat{B} C=P \hat{Q} R$, then $\triangle A B C$ is similar to $\triangle P Q R$ (SAS Similarity Test).


Angle Properties of Circles
Angles in the same segment are equal, i.e. $\angle A P B=\angle A Q B$.


## Tuition

An angle at the centre of a circle is twice that of any angle at the circumference subtended by the same arc, i.e. $\angle A O B=2 \times \angle A P B$.


Angles in opposite segments are supplementary, i.e. $\angle a+\angle c=180^{\circ} ; \angle b+\angle d=180^{\circ}$.


An angle in a semicircle is always equal to $90^{\circ}$, i.e. $A O B$ is a diameter $\Leftrightarrow \angle A P B=90^{\circ}$.


## Tuition

## A-Math Formula List - Additional Math (4049)

## Differentiation

| $\frac{d y}{d x}\left(a x^{n}\right)=a n x^{n-1}$ | Differentiate Constant | Sum / Difference of Function |
| :--- | :--- | :--- |
| Where' $a^{\prime}$ and ' $n$ ' | $\frac{d y}{d x}(a)=0$ | $\frac{d}{d x}(u \pm v)=\frac{d u}{d x} \pm \frac{d v}{d x}$ |
| are constants. . |  | Quotient Rule |
| Chain Rule | $\frac{d}{d x}\left(\frac{u}{v}\right)=\frac{v \frac{d u}{d x}-u \frac{d v}{d x}}{v^{2}}$ |  |
| $\frac{d y}{d x}=\frac{d y}{d u} \times \frac{d u}{d x}$ |  |  |

Product Rule

$$
\frac{d}{d x}(u v)=u \frac{d v}{d x}+v \frac{d u}{d x}
$$

Differentiation of Trigonometry

$$
\frac{d}{d x}(\sin x)=\cos x \quad \frac{d}{d x}(\tan x)=\sec ^{2} x \quad \frac{d}{d x}(\cos x)=-\sin x
$$

Use Chain Rule to differentiate the functions below.

$$
\begin{aligned}
& \frac{d}{d x}[a \sin (b x+c)]=a \times \cos (b x+c) \times b \\
& \frac{d}{d x}[a \cos (b x+c)]=a \times-\sin (b x+c) \times b
\end{aligned}
$$

## Tuition

| $\frac{d}{d x}[a \tan (b x+c)]=a \times \sec ^{2}(b x+c) \times b$ |  |
| :--- | :--- |
| $\frac{d}{d x}\left[a \sin ^{n}(b x+c)\right]=a \times n \times \sin ^{n-1}(b x+c) \times \cos (b x+c) \times b$ |  |
| $\frac{d}{d x}\left[a \cos ^{n}(b x+c)\right]=a \times n \times \cos ^{n-1}(b x+c) \times-\sin (b x+c) \times b$ |  |
| $\frac{d}{d x}\left[a t a n^{n}(b x+c)\right]=a \times n \times \tan ^{n-1}(b x+c) \times \sec ^{2}(b x+c) \times b$ |  |
| Exponential/Natural Logarithm Function |  |
| $\frac{d}{d x}\left(e^{a x+b}\right)=a e^{a x+b}$ | $\frac{d}{d x}\left(e^{x}\right)=e^{x}$ |
| 'a $^{\prime}$ is $a \operatorname{constant}$ | $\frac{d}{d x}[\ln (a x+b)]=\frac{a}{a x+b}$ <br> $\frac{d}{d x}(\ln x)=\frac{1}{x}($ where $\times \times 0)$ <br> $($ where $a x+b \times 0)$ |

## Tuition

## A-Math Formula List - Additional Math (4049)

| Increasing Function <br> If $y$ is an increasing function ( $y$ i cheases as $x$ increases), the gradient is positive. <br> ie. $\frac{d y}{d r}>0$. <br> e.g. | Decreasing Function <br> If $y$ is a dercrasing funcion (y decreases as $x$ increase), tre sradien is negative, <br> ie. $\frac{d y}{d x}<0$. |
| :---: | :---: |
| Turning \& Inflexion Points | Note: <br> When $\frac{d^{2} y}{d x^{2}}=0$ it does not necessarily mean that it is a point of inflexion. There are some graphs that will show $\frac{d^{2} y}{d x^{2}}=0$ even though they are turning points (maximum or minimum point). You will then need to carry out a $1^{\text {st }}$ order derivative test. |

## Tuition

## A-Math Formula List - Additional Math (4049)

$1^{\text {st }}$ order derivative test


Point of inflexion

|  | $x^{2}$ | $x_{0}$ | $x^{*}$ |  |
| :---: | :---: | :---: | :---: | :---: |
| $\frac{\mathrm{~d} y}{\mathrm{~d} x}$ | $>0$ | 0 | $>0$ |  |
| slope | 1 | - | 1 |  |
| stationary <br> point |  |  |  |  |

Minimum point
Minimum point

|  | $x$ | $x_{0}$ | $x^{*}$ |  |
| :---: | :---: | :---: | :---: | :---: |
| $\frac{\mathrm{~d} y}{\mathrm{~d} x}$ | $<0$ | 0 | $>0$ |  |
| slope | 1 | - | 1 |  |
| stationary <br> point |  |  |  |  |

Point of inflexion

| Point of inflexion |  |  |  |
| :---: | :---: | :---: | :---: |
|  | $x$ | $x_{0}$ | $x^{+}$ |
| $\frac{\mathrm{d} y}{\mathrm{~d} x}$ | $<0$ | 0 | $<0$ |
| slope | 1 | - | 1 |
| stationary <br> point |  |  |  |

## Integration

| $\int a x^{n} d x=\frac{\alpha x^{n+1}}{n+1}+c$ <br> where $n \neq-1$ | Integrate Constant $\int a d x=a x+c$ | Productr $\int(a x+b)$ | $n \neq 1, a \neq 0 ; a \& b$ are constants $x=\frac{(a x+b)^{n+1}}{(n+1)(a)}+c$ |
| :---: | :---: | :---: | :---: |
| Product of constant and a $\int a f(x) d x=a \int f$ | unction <br> $x) d x$ | Sum and D $\int[\alpha f(x) \pm$ | erence of function <br> $x)] d x=\alpha \int f(x) d x \pm \beta \int g(x) d x$ |
| Integration of Trigonometry |  |  |  |
| $\int \cos x d x=\sin x+c$ | $\int \sin x d x=-\cos x+c$ |  | $\int \sec ^{2} x d x=\tan x+c$ |
| $\int a \cos b x d x=\frac{a \sin b x}{b}+c$ | $\int a \sin b x d x=\frac{-a \cos b x}{b}+c$ |  | $\int a \sec ^{2} b x d x=\frac{a \tan b x}{b}+c$ |

## Tuition

A-Math Formula List - Additional Math (4049)

## Integration of exponential and Ln

| $\int \frac{1}{x} d x=\ln x+c$ where $\times>0$ | $\int \frac{1}{a x+b} d x=\frac{\ln (a x+b)}{a}+C$ |  |
| :--- | :--- | :--- |
|  | $\int \frac{1}{a x^{n}+b} d x=\frac{\ln \left(a x^{n}+b\right)}{a \times n \times x^{n-1}}+C$ |  |
| $\int e^{x} d x=e^{x}+c$ | $\int e^{a x+b} d x=\frac{e^{a x+b}}{a}+C$ |  |
|  | $\int e^{a x^{n}+b} d x=\frac{e^{a x^{n}+b}}{a \times n \times x^{n-1}}+C$ |  |
| Basic Properties of Definite Integral |  |  |
| $\int_{a}^{a} f(x) d x=0$ | $\int_{a}^{b} f(x) d x=-\int_{b}^{a} f(x) d x$ | $\int_{a}^{b} f(x) d x=\int_{c}^{b} f(x) d x+\int_{a}^{c} f(x) d x$ |

## Integration of Area



## Tuition

A-Math Formula List - Additional Math (4049)


## Kinematics



