Section A (52 marks)

Answer **all** the questions in this section.

1 Lee records the number of texts he receives each week for 30 weeks.
The stem-and-leaf diagram shows his results.

2	7
3	5 8 9 9
4	0 3 5 6 8
5	2 4 5 5 6 7 8 8 9 9
6	1 3 3 7 7 7 8
7	0 3 3

Key 2 | 7 means 27

Find

(a) the number of weeks in which Lee receives fewer than 45 texts,

Answer 7 [1]

(b) the median.

$\frac{56+57}{2} = 56.5$

Answer 56.5 [1]

2 The sizes of the three angles in a triangle are in the ratio 7 : 3 : 2.

Find the three angles of the triangle.

$\frac{180°}{7+3+2} \times 2 = 30°$

$\frac{180}{7+3+2} \times 3 = 45°$

$\frac{180}{7+3+2} \times 7 = 105°$

Answer 30° , 45° , 105° [3]

3 **(a)** Calculate

(i) $\sqrt[3]{0.000\,3689 + 0.004\,529}$, $= 0.1698256575$

Answer 0.170 (3sf) [1]

(ii) $\dfrac{5.63^2}{8732 - 235.6}$. $= 0.00373062$

Answer 0.00373 (3sf) [1]

(b) The populations of the four main islands in Japan are shown below.

Island	Population (millions)
Honshu	104.0
Hokkaido	5.348
Kyushu	12.97
Shikoku	3.797

Calculate the total population of the four islands.
Give your answer in standard form.

total $= 104.0 + 5.348 + 12.97 + 3.797$
$= 126.115$ million $= 126.115 \times 10^6$

Answer 1.26115×10^8 [2]

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4 **(a)** Simplify $5(x-1) - 2(x+2)$.

$5x - 5 - 2x - 4 = 3x - 9$

Answer $3x - 9$ [2]

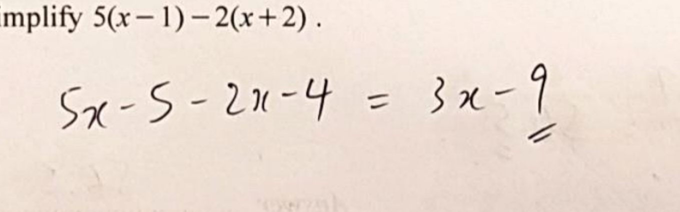
(b) **(i)** Solve the inequality $5m < 21$.

$5m < 21$

$m < 4\frac{1}{5}$

Answer $m < 4\frac{1}{5}$ [1]

(ii) Write down the largest integer value for m.



Answer $m =$ 4 [1]

5 Written as the product of its prime factors, $2520 = 2^3 \times 3^2 \times 5 \times 7$.

(a) Write 2160 as the product of its prime factors.

$$
\begin{array}{r|l}
2 & 2160 \\
2 & 1080 \\
2 & 540 \\
2 & 270 \\
3 & 135 \\
3 & 45 \\
3 & 15 \\
5 & 5 \\
 & 1
\end{array}
$$

$2160 = 2^4 \times 3^3 \times 5$

Answer $2^4 \times 3^3 \times 5$ [2]

(b) Giving each answer as the product of its prime factors,

(i) find the highest common factor (HCF) of 2160 and 2520,

$2520 = 2^3 \times 3^2 \times 5 \times 7$

$2160 = 2^4 \times 3^3 \times 5 \times 1$

$HCF = 2^3 \times 3^2 \times 5 \times 1$

Answer $2^3 \times 3^2 \times 5$ [1]

(ii) find the smallest positive integer value of *a* such that 2520*a* is a perfect cube.

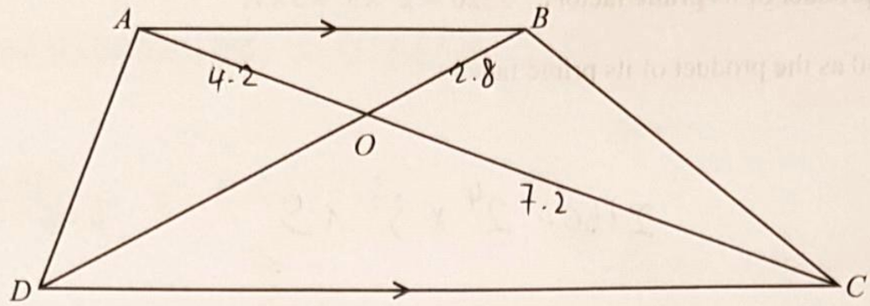
$2520a = 2^3 \times 3^2 \times 5 \times 7 \times a$

$= (2 \times 3 \times 5 \times 7)(2 \times 3 \times 5 \times 7)(2 \times 3 \times 5 \times 7)$

$a = 3 \times 5^2 \times 7^2$

Answer $a =$ $3 \times 5^2 \times 7^2$ [1]

6



In quadrilateral *ABCD*, *AB* is parallel to *DC*.

(a) What is the mathematical name of quadrilateral *ABCD*?

Answer Trapezium. [1]

(b) Given that triangle *AOB* is similar to triangle *COD*, identify all the pairs of angles in triangle *AOB* and triangle *COD* that are equal.

Answer

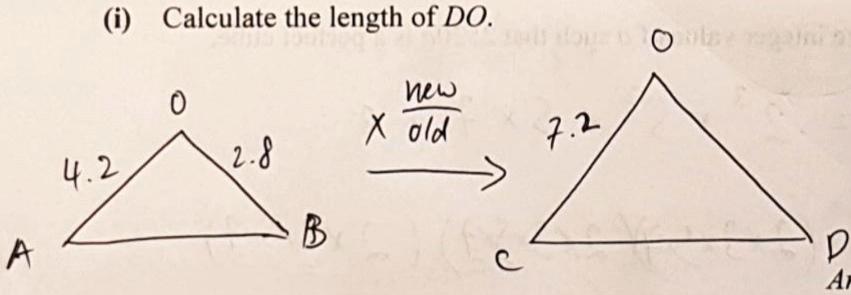
$\angle OAB = \angle OCD$

$\angle OBA = \angle ODC$

$\angle AOB = \angle COD$. [2]

(c) $AO = 4.2$ cm, $BO = 2.8$ cm and $CO = 7.2$ cm.

(i) Calculate the length of *DO*.



$\frac{new}{old} = \frac{7.2}{4.2} = \frac{12}{7}$

$DO = 2.8 \times \frac{12}{7} = 4.8$

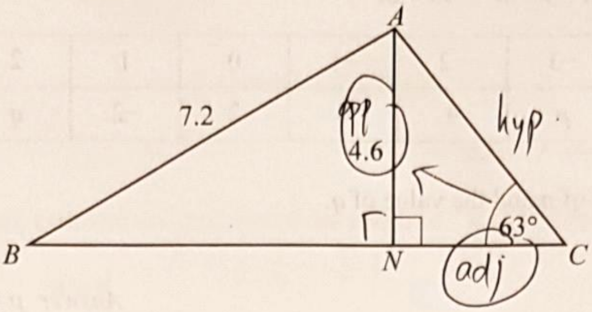
Answer 4.8 cm [2]

(ii) Find the scale factor for the reduction of triangle *COD* to triangle *AOB*.

$\frac{small}{big} = \frac{4.2}{7.2} = \frac{7}{12}$.

Answer $\frac{7}{12}$ [1]

7



ABC is a triangle and AN is perpendicular to BC.
$AB = 7.2$ cm, $AN = 4.6$ cm and angle $ACN = 63°$.

(a) Calculate

(i) BN,

By PT, $7.2^2 = 4.6^2 + BN^2$

$BN^2 = 7.2^2 - 4.6^2$

$BN^2 = 30.68$

$BN = 5.5389$

$= 5.54$ cm (3sf)

Answer $BN =$ ....5.54.... cm [3]

(ii) NC.

$\tan 63° = \frac{4.6}{NC}$

$NC = \frac{4.6}{\tan 63°} = 2.3438$

$= 2.34$ cm (3sf)

Answer $NC =$ ....2.34.... cm [3]

(b) Calculate the area of triangle ABC.

$BC = 5.5389 + 2.3438$

$= 7.88277$ cm

area $= \frac{1}{2} \times 7.88277 \times 4.6$

$= 18.130$

$= 18.1$ cm^2 (3sf)

Answer ....18.1.... cm^2 [2]

8 This table of values is for $y = x^3 - 5x + 2$.

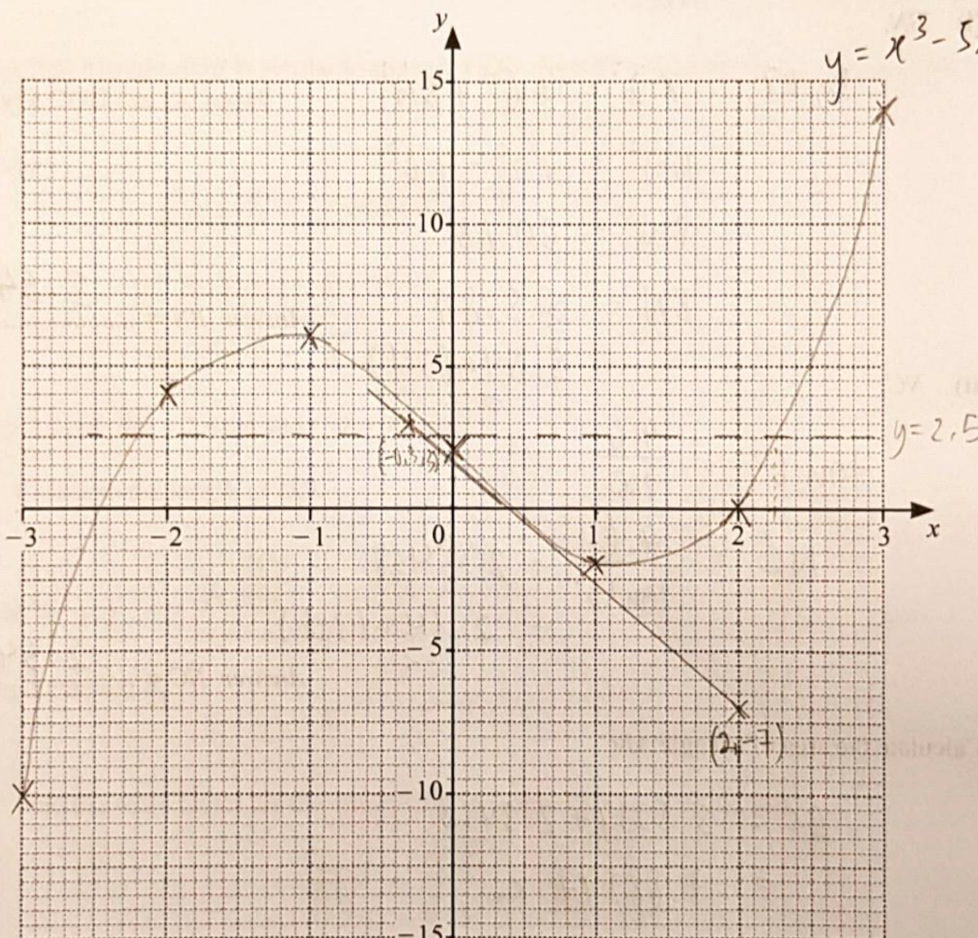
x	-3	-2	-1	0	1	2	3
y	p	4	6	2	-2	q	14

(a) Calculate the value of p and the value of q.

Answer $p =$ .... -10

$q =$ .... 0 [2]

(b) Draw the graph of $y = x^3 - 5x + 2$ for $-3 \leqslant x \leqslant 3$.



[3]

$\frac{dy}{dx} = 3x^2 - 5$

at $x = 0.5$, gradient $= -4.25$

$y + \frac{3}{8} = -4.25(x - 0.5)$

$y = -4.25x + 1.75$

(c) Use your graph to find the largest value of x when $y = 2.5$.

Answer $x =$ 2.25 [1]

(d) By drawing a tangent, estimate the gradient of the graph of $y = x^3 - 5x + 2$ when $x = 0.5$.

$(2, -7)$ and $(-0.3, 3)$

$$\text{gradient} = \frac{3-(-7)}{-0.3-2}$$

$$= -4.3478$$

$$= -4.35 \;//\; (3sf)$$

Answer −4.35 [2]

9 (a) Factorise completely $4x^2y - 6xy^3$.

$2xy(2x-3y^2)$ //

Answer ~~2xy~~ $2xy(2x-3y^2)$ [2]

(b) Rearrange this equation to make b the subject.

$$4b - 3 = \frac{b+2}{a}$$

$$a(4b-3) = b+2$$

$$4ab - 3a = b+2$$

$$4ab - b = 3a+2$$

$$b(4a-1) = 3a+2$$

$$b = \frac{3a+2}{4a-1} \;//$$

Answer $b = \dfrac{3a+2}{4a-1}$ [3]

(a) Calculate the total number of tourists from China and Australia who visited Singapore in 2018.

Write your answer to the nearest million.

2106000 + 1044000 = 3150 000

= 3 million

Answer 3000 000 [1]

(b) Calculate the total amount spent by tourists from China in 2018 to the nearest million.

mean × no. of tourists

1205 × 2106 000 = 2537 7 3 0000

= 2538 million

Answer $ 2538 million [2]

(c) Based on the 2018 data, a tourist from Australia is expected to spend $979 in total during their stay. How much would a tourist from Australia spend per day during their stay?

$\frac{979}{3.6}$ = $271.94 (2dp).

Answer $ 271.94 [1]

(d) A café focuses its marketing on tourists from one country only, either China or Australia.

The café decides which country to focus on by calculating the average amount spent per tourist during their stay on food and beverages.

Based on the 2017 data, a tourist from China spent more on food and beverages than a tourist from Australia. The café therefore focused its marketing on tourists from China.

Based on the 2018 data, should the café continue to focus its marketing on Chinese tourists? Show working to support your answer.

Answer In 2018, average per Chinese tourist = $1205.

food and beverage spending (China) = $1205 \times \frac{9}{100}$

= $108.45.

In 2018, average per Australian tourist = $979.

food and beverage spending = $979 \times \frac{16}{100}$

= $156.64.

Since the average Australian spends more, they should not focus ~~on~~ its marketing on Chinese tourists. [4]

Section B (8 marks)

Answer **one** question from this section. Each question carries 8 marks.

11 Farmer A has 120 cows.
He records each cow's milk production, L litres, for one week.
The results are shown in the table.

Amount of milk (L litres)	$150 < L \leqslant 160$	$160 < L \leqslant 170$	$170 < L \leqslant 180$	$180 < L \leqslant 190$	$190 < L \leqslant 200$
Number of cows (frequency)	12	32	43	24	9

155 165 175 185 195

(a) **(i)** Calculate an estimate of the mean and standard deviation of the amount of milk produced by each cow.

From calculator,

$\bar{x} = 173.833$

$= 174 l$ (3sf)

$\sigma_x = 10.738$

$= 10.7 l$ (3sf)

Answer Mean ....174........ litres

Standard deviation ....10.7.... litres [3]

(ii) Farmer B also records the milk production of each of his cows in the same week.
The mean production for each of farmer B's cows is 179 litres and the standard deviation is 9.2 litres.

Which farmer, A or B, has cows with more consistent milk production?
Give a reason for your answer.

$\downarrow \sigma x$

Answer

Farmer B, because the standard deviation is lower than the standard deviation of Farmer A's cows.

[1]

(b) Two of Farmer A's cows are chosen at random.

Calculate the probability that

(i) both cows had a production of more than 180 litres in the week,

$\frac{33}{120}$ more than 180 — $\frac{32}{119}$ more than 180; $\frac{87}{119}$ less than 180.

~~less than 180~~

$$= \frac{33}{120} \times \frac{32}{119}$$

$$= \frac{44}{595}$$

Answer ....$\frac{44}{595}$.... [2]

(ii) at least one of the cows had a production of more than 160 litres in the week.

$\frac{108}{120}$ more than 160 — $\frac{107}{119}$ more than 160 $= \frac{108}{120} \times \frac{107}{119} = \frac{963}{1190}$

$\frac{12}{119}$ less than 160 $= \frac{108}{120} \times \frac{12}{119} = \frac{54}{595}$

$\frac{12}{120}$ less than 160 — $\frac{108}{119}$ more than 160 $= \frac{12}{120} \times \frac{108}{119} = \frac{54}{595}$

$\frac{11}{119}$ less than 160 X

$$\frac{1179}{1190}$$

Answer ....$\frac{1179}{1190}$.... [2]

OR

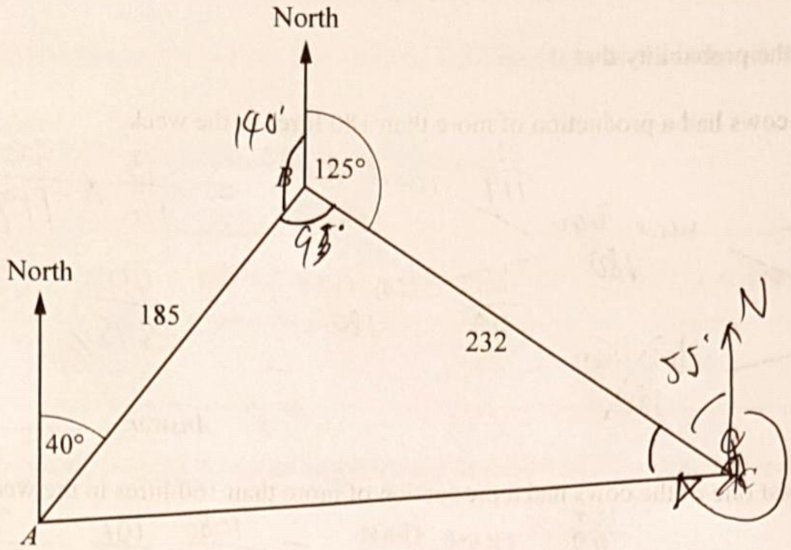
$$P(\text{both less than } 160\ell) = \frac{12}{120} \times \frac{11}{119}$$

$$= \frac{11}{1190}$$

$$P(\text{at least one}) = 1 - \frac{11}{1190}$$

$$= \frac{1179}{1190}$$

12



A ship sails 185 km on a bearing of 040° from *A* to *B*.
It then sails 232 km on a bearing of 125° from *B* to *C*.

(a) Show that angle $ABC = 95°$.

Answer

$180° - 40° = 140°$ (int ∠)

$\angle ABC = 360° - 140° - 125°$ (∠s at a pt).

$= 95°$ (shown)

[1]

(b) Calculate the distance *AC*.

$AC^2 = 185^2 + 232^2 - 2(185)(232)\cos 95°$

$AC^2 = 95530.4489$

$AC = 309.0800$

$= 309$ km (3sf)

Answer $AC =$ 309 km [3]

* 0011297251015 *

(c) Calculate the bearing of *A* from *C*.

$$\frac{\sin \angle ACB}{185} = \frac{\sin 95^\circ}{309.08}$$

$$\angle ACB = \sin^{-1}\left(\frac{\sin 95^\circ}{309.08} \times 185\right)$$

$$= 36.6034^\circ$$

$$180^\circ - 125^\circ = \cancel{6}\ 55^\circ \text{ (int } \angle\text{)}$$

$\therefore$ bearing of A from C $= 360^\circ - 55^\circ - 36.6034^\circ$ ($\angle$s at a pt)

$= 268.396$

$= 268.4^\circ$ (1dp)

Answer 268.4° [4]