

Content

| No. | Topic / sub topic | Page |
|-----|--|------|
| 1 | Natural numbers | 1 |
| 2 | Whole Numbers | 1 |
| 3 | Integers | 1 |
| 4 | Rational Numbers | 1 |
| 5 | Irrational Numbers | 1 |
| 6 | Terminating Decimals | 1 |
| 7 | Recurring Decimals | 1 |
| 8 | Significant figures | 1 |
| 9 | Decimal Places | 1 |
| 10 | Standard Form | 1 |
| 11 | Conversion Factors | 2 |
| 12 | Time | 2 |
| 13 | Percentages | 2 |
| 14 | Simple Interest | 2 |
| 15 | Compound Interest | 2 |
| 16 | Speed, Distance and Time | 3 |
| 17 | Quadratic Equations | 3 |
| 18 | Expansion of algebraic expressions | 3 |
| 19 | Factorization of algebraic expressions | 3 |
| 20 | Ordering | 3 |
| 21 | Variation | 4 |
| 22 | PYTHAGORAS' THEOREM | 4 |
| 23 | Area and Perimeter | 4 |
| 24 | Surface Area and Volume | 5 |
| 25 | Angles on a straight line | 6 |
| 26 | Vertically opposite angles | 6 |
| 27 | Different types of triangles | 6 |
| 28 | Parallel Lines | 6 |
| 29 | Types of angles | 6 |
| 30 | Angle properties of triangle | 7 |
| 31 | Congruent Triangles | 7 |
| 32 | Similar Triangles | 7 |
| 33 | Areas of Similar Triangles | 8 |
| 34 | Polygons | 8 |
| 35 | Similar Solids | 8 |
| 36 | CIRCLE | 9 |
| 37 | Chord of a circle | 9 |
| 38 | Tangents to a Circle | 10 |
| 39 | Laws of Indices | 10 |
| 40 | Solving Inequalities | 11 |
| 41 | TRIGONOMETRY | 11 |
| 42 | Bearing | 11 |
| 43 | Cartesian co-ordinates | 12 |
| 44 | Distance – Time Graphs | 12 |
| 45 | Speed – Time Graphs | 12 |
| 46 | Velocity | 13 |
| 47 | Acceleration | 13 |
| 48 | SETS | 13 |

| | | |
|----|----------------------------|--------|
| 49 | Loci and construction | 14 |
| 50 | Vectors | 14 |
| 51 | Column Vectors | 15 |
| 52 | Parallel Vectors | 15 |
| 53 | Modulus of a Vector | 15 |
| 54 | MATRICES | 15 |
| 55 | The Inverse of a Matrix | 15 |
| 56 | Transformations | 16 -17 |
| 57 | Transformation by Matrices | 18 |
| 58 | STATISTICS | 19 |
| 59 | Probability | 20 |
| 60 | Symmetry | 21 |

Conversion Factors:

Length:

1 km = 1000 m
1 m = 100 cm
1 cm = 10 mm

km means kilometer
m means meter
cm means centimeter
mm means millimeter

Mass:

1 kg = 1000 gm where kg means kilogram
1 gm = 1000 mgm gm means gram
1 tonne = 1000 kg mgm means milligram

Volume:

1 litre = 1000 cm³
1 m³ = 1000 litres
1 kilo litre = 1000 litre
1 dozen = 12

Time:

1 hour = 60 minutes = 3600 seconds
1 minute = 60 seconds.
1 day = 24 hours
1 year = 12 months
 = 52 weeks
 = 365.25 days.

1 week = 7 days
1 leap year = 366 days
1 light year = 9.46×10^{12} km.

Percentages:

- Percent means per hundred.
- To express one quantity as a percentage of another, first write the first quantity as a fraction of the second and then multiply by 100.
- Profit = S.P. – C.P.
- Loss = C.P. – S.P.
- Profit percentage = $\frac{SP-CP}{CP} \times 100$
- Loss percentage = $\frac{CP-SP}{CP} \times 100$

where CP = Cost price and SP = Selling price

Simple Interest:

To find the interest:

- $i = \frac{PRT}{100}$ where
P = money invested or borrowed
R = rate of interest per annum
T = Period of time (in years)

To find the amount:

- $A = P + I$ where A = amount

Compound Interest:

$$A = p \left(1 + \frac{r}{100} \right)^n$$

Where,

A stands for the amount of money accruing after *n* year.

P stands for the principal

R stands for the rate per cent per annum

n stands for the number of years for which the money is invested.

Variation:

Direct Variation:

y is proportional to x

$$y \propto x$$

$$y = kx$$

Inverse Variation:

y is inversely proportional to x

$$y \propto \frac{1}{x}$$

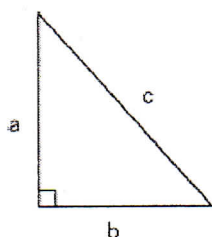
$$y = \frac{k}{x}$$

MENSURATION

PYTHAGORAS' THEOREM

For all the **right angled** triangles "the square on the hypotenuse is equal to the sum of the squares on the other two sides"

$$c^2 = a^2 + b^2$$



$$c = \sqrt{a^2 + b^2}$$

$$b = \sqrt{c^2 - a^2}$$

$$a = \sqrt{c^2 - b^2}$$

Area and Perimeter:

| Figure | Diagram | Area | Perimeter |
|---------------|--|--|---|
| Rectangle | <p>A rectangle with a horizontal side labeled 'l' and a vertical side labeled 'b'.</p> | Area = $l \times b$ | perimeter = $2(l + b)$ |
| Square | <p>A square with all four sides labeled 'a'.</p> | Area = side \times side = $a \times a$ | perimeter = $4 \times$ side = $4 \times a$ |
| Parallelogram | <p>A parallelogram with a horizontal base labeled 'b', a slanted side labeled 'a', and a vertical height line labeled 'h' drawn from the top side to the base. The angle between side 'a' and base 'b' is labeled θ.</p> | Area = $b \times h$ Area = $ab \sin \theta$ where a, b are sides and θ is the included angle | perimeter = $2(a + b)$ |
| Triangle | <p>A triangle with vertices labeled A, B, and C. The base AC is labeled 'b'. A vertical height line from vertex B to the base is labeled 'h'. The sides AB and BC are labeled 'c' and 'a' respectively.</p> | Area = $\frac{1}{2} \times \text{base} \times \text{height}$ Area = $\frac{1}{2} ab \sin C$ $= \sqrt{s(s-a)(s-b)(s-c)}$ where $s = \frac{a+b+c}{2}$ | perimeter = $a + b + c$ |

GEOMETRY

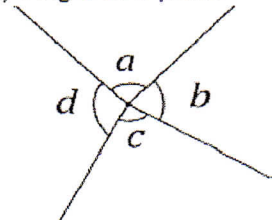
(a) Angles on a straight line

The angles on a straight line add up to 180° .

$$x + y + z = 180^\circ$$



(b) Angle at a point



The angles at a point add up to 360° .

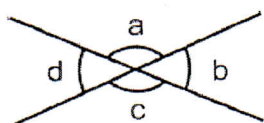
$$a + b + c + d = 360^\circ$$

(c) Vertically opposite angles

If two straight lines intersect, then

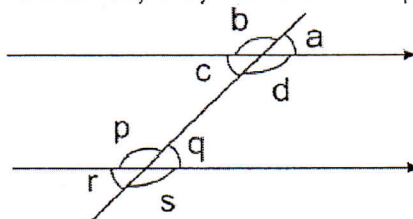
$$a = c$$

$$b = d \text{ (Vert, opp. } \angle\text{s)}$$



Parallel Lines:

When lines never meet, no matter how far they are extended, they are said to be parallel.

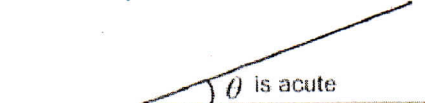


- Vertically opposite angles are equal.
 $a = c$; $b = d$; $p = r$ and $q = s$
- Corresponding angles** are equal.
 $a = p$; $b = q$; $c = r$ and $d = s$
- Alternate angles** are equal.
 $c = q$ and $d = p$.
- Sum of the angles of a triangle is 180° .
- Sum of the angles of a quadrilateral is 360° .

Types of angles

Given an angle θ , if

$\theta < 90^\circ$, then θ is an **acute** angle



$90^\circ < \theta < 180^\circ$, then θ is an **obtuse** angle



$180^\circ < \theta < 360^\circ$, then θ is an **reflex** angle



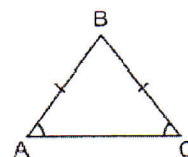
Triangles

Different types of triangles:

1. An isosceles triangle has 2 sides and 2 angles the same.

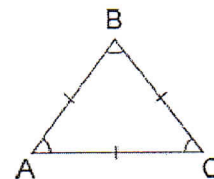
$$AB = AC$$

$$ABC = BCA$$



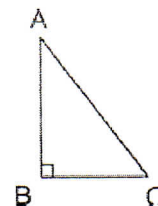
2. An equilateral triangle has 3 sides and 3 angles the same.

$$AB = BC = CA \text{ and } ABC = BCA = CAB$$

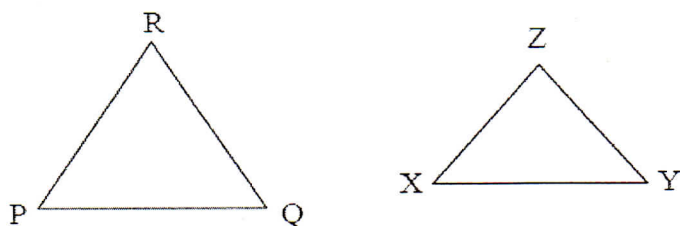


3. A triangle in which one angle is a right angle is called the right angled triangle.

$$ABC = 90^\circ$$



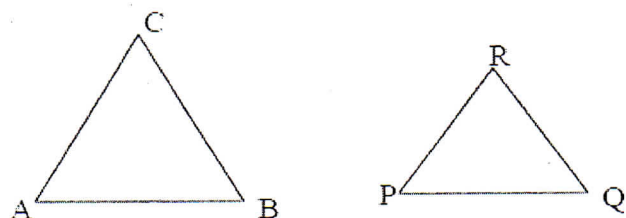
iii) The ratios of the corresponding sides are equal and the angles between them are equal.



ΔPQR is similar to ΔXYZ (if, for eg: $\angle P = \angle X$ and $\frac{PQ}{XY} = \frac{PR}{XZ}$)

Areas of Similar Triangles:

The ratio of the areas of similar triangles is equal to the ratio of the square on corresponding sides.



$$\frac{\text{area of } \Delta ABC}{\text{area of } \Delta PQR} = \frac{AB^2}{PQ^2} = \frac{BC^2}{QR^2} = \frac{AC^2}{PR^2}$$

Polygons:

- The exterior angles of a polygon add up to 360° .
- The sum of the interior angles of a polygon is $(n - 2) \times 180^\circ$ where n is the number of sides of the polygon.
- A regular polygon has equal sides and equal angles.
- If the polygon is regular and has n sides, then each exterior angle $= \frac{360}{n}$

| | | |
|--------------------|-------------------------|--------------------|
| 3 sides = triangle | 4 sides = quadrilateral | 5 sides = pentagon |
| 6 sides = hexagon | 7 sides = heptagon | 8 sides = octagon |
| 9 sides = nonagon | 10 sides = decagon | |

Similar Solids:

If two objects are similar and the ratio of corresponding sides is k , then

- the ratio of their areas is k^2 .
- the ratio of their volumes is k^3 .

Length

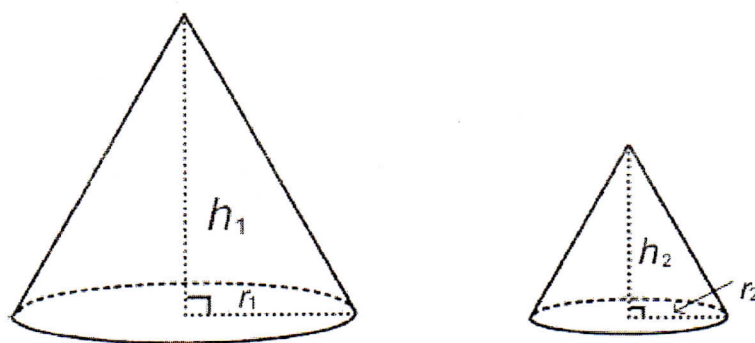
$$\frac{l_1}{l_2} = \frac{r_1}{r_2} = \frac{h_1}{h_2}$$

Area

$$\frac{A_1}{A_2} = \frac{r_1^2}{r_2^2} = \frac{h_1^2}{h_2^2}$$

Volume

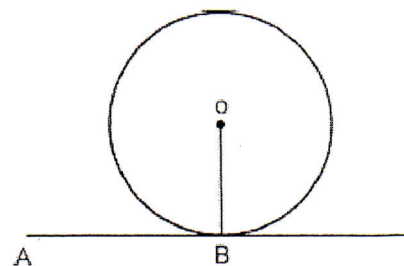
$$\frac{V_1}{V_2} = \frac{r_1^3}{r_2^3} = \frac{h_1^3}{h_2^3}$$



Tangents to a Circle:

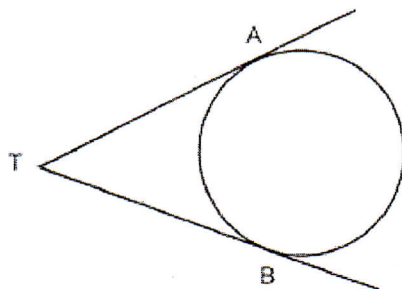
- The angle between a tangent and the radius drawn to the point of contact is 90° .

$$\angle ABO = 90^\circ$$



- From any point outside a circle just two tangents to the circle may be drawn and they are of equal length.

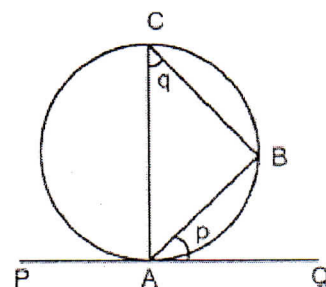
$$TA = TB$$



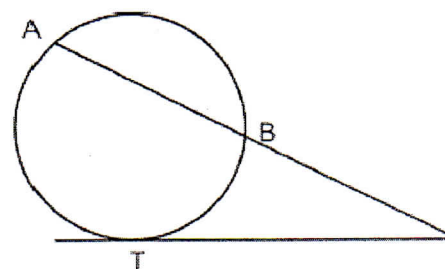
Alternate Segment Theorem

The angle between a tangent and a chord through the point of contact is equal to the angle subtended by the chord in the alternate segment.

$$\angle QAB = \angle ACB \quad (p = q)$$



- $TC^2 = AC \times BC$



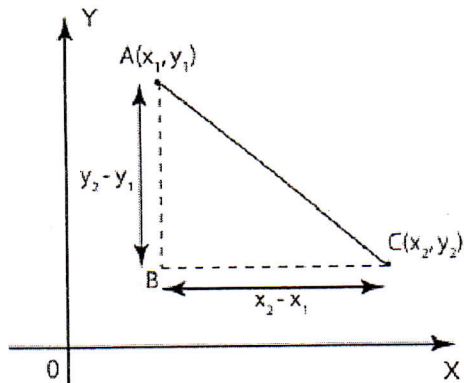
INDICES:

- $a^m \times a^n = a^{m+n}$
- $a^m \div a^n = a^{m-n}$
- $(a^m)^n = a^{mn}$
- $a^0 = 1$
- $a^{-n} = \frac{1}{a^n}$
- $(a \times b)^m = a^m \times b^m$

- $\left(\frac{a}{b}\right)^m = \frac{a^m}{b^m}$
- $(\sqrt[n]{a})^m = a^{m/n}$
- $\sqrt{a} \times \sqrt{b} = \sqrt{a \times b}$
- $\sqrt{\frac{a}{b}} = \frac{\sqrt{a}}{\sqrt{b}}$

$$(\sqrt{a})^2 = a$$

Distance and Gradient



Distance Between Point A and C =

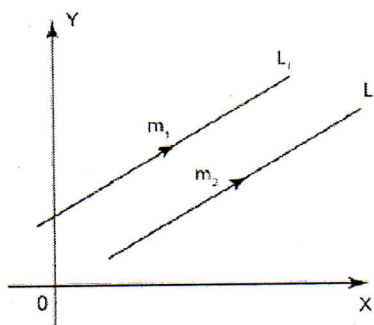
$$\sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$$

Gradient of line AC, $m = \frac{y_2 - y_1}{x_2 - x_1}$

Or

Gradient of a line, $m = -\left(\frac{y - \text{intercept}}{x - \text{intercept}}\right)$

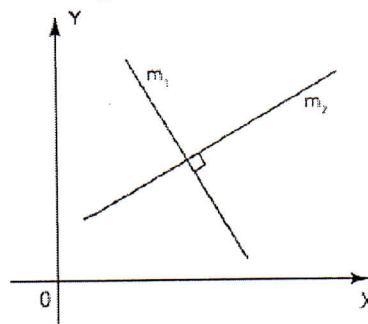
Parallel Lines



When 2 lines are parallel,

$$m_1 = m_2$$

Perpendicular Lines



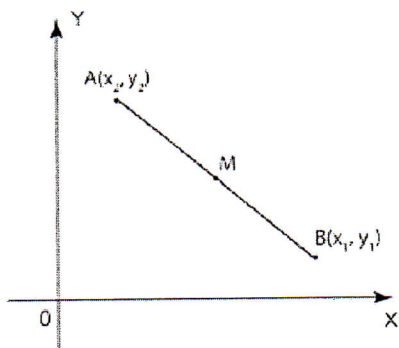
When 2 lines are perpendicular to each other,

$$m_1 \times m_2 = -1$$

m_1 = gradient of line 1

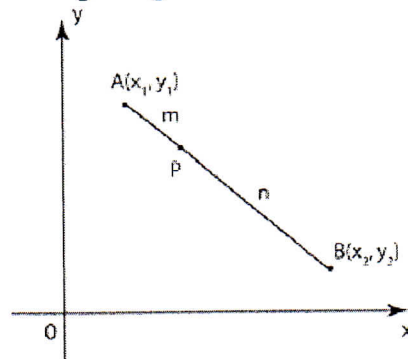
m_2 = gradient of line 2

Midpoint



Midpoint, $M = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$

A point dividing a segment of a line



A point dividing a segment of a line

$$P = \left(\frac{nx_1 + mx_2}{m + n}, \frac{ny_1 + my_2}{m + n}\right)$$

Velocity:

Velocity is the rate of change of distance with respect to the time.

Acceleration:

Acceleration is the rate of change of velocity with respect to time.

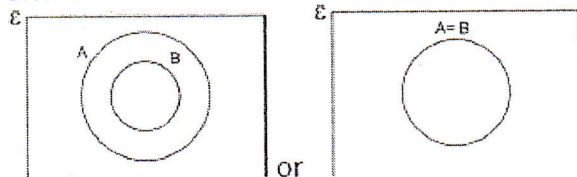
SETS:

Notations

- ξ = universal set
- \cup (union) = all the elements
- \cap (intersection) = common elements
- \emptyset or $\{ \}$ = empty set
- \in = belongs to
- \notin = does not belongs to
- \subseteq = Subset
- A' = compliment of A (i.e. the elements of ξ - the elements of A)
- $n(A)$ = the number of elements in A.
- De Morgan's Laws: $(A \cup B)' = (A' \cap B')$
 $(A \cap B)' = (A' \cup B')$

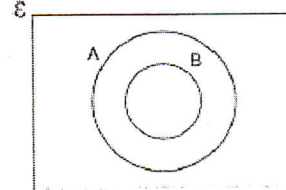
Subset \subseteq

$B \subseteq A$ means every elements of set B is also an element of set A.



Proper subset \subset

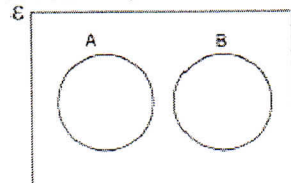
$B \subset A$ means every element of B is an element of set A but $B \neq A$.



Disjoint sets

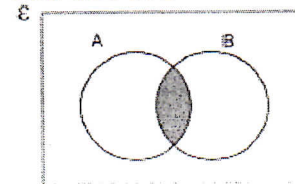
Disjoint set do not have any element in common. If A and B are disjoint sets, then

$$A \cap B = \emptyset$$



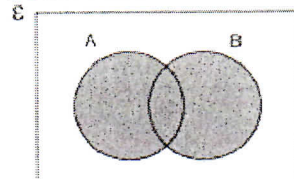
Intersection \cap

$A \cap B$ is the set of elements which are in A and also in B



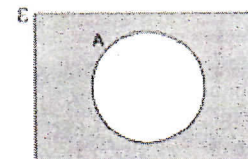
Union \cup

$A \cup B$ is the set of elements in either A, B or both A and B.



Complement

The complement of A, written as A' refers to the elements in ξ but not in A.



Column Vectors:

The top number is the horizontal component and the bottom number is the vertical component.

$$\begin{pmatrix} x \\ y \end{pmatrix}$$

Parallel Vectors:

- Vectors are parallel if they have the same direction. Both components of one vector must be in the same ratio to the corresponding components of the parallel vector.
- In general the vector $k \begin{pmatrix} a \\ b \end{pmatrix}$ is parallel to $\begin{pmatrix} a \\ b \end{pmatrix}$

Modulus of a Vector:

The modulus of a vector a , is written as $|a|$ and represents the length (or magnitude) of the vector.

In general, if $x = \begin{pmatrix} m \\ n \end{pmatrix}$, $|x| = \sqrt{m^2 + n^2}$

MATRICES:

Addition and Subtraction:

Matrices of the same order are added (or subtracted) by adding (or subtracting) the corresponding elements in each matrix.

$$\begin{pmatrix} a & b \\ c & d \end{pmatrix} + \begin{pmatrix} p & q \\ r & s \end{pmatrix} = \begin{pmatrix} a+p & b+q \\ c+r & d+s \end{pmatrix}$$

$$\begin{pmatrix} a & b \\ c & d \end{pmatrix} - \begin{pmatrix} p & q \\ r & s \end{pmatrix} = \begin{pmatrix} a-p & b-q \\ c-r & d-s \end{pmatrix}$$

Multiplication by a Number:

Each element of a matrix is multiplied by the multiplying number.

$$k \times \begin{pmatrix} a & b \\ c & d \end{pmatrix} = \begin{pmatrix} ka & kb \\ kc & kd \end{pmatrix}$$

Multiplication by another Matrix:

Matrices may be multiplied only if they are compatible. The number of columns in the left-hand matrix must equal the number of rows in the right-hand matrix.

$$\begin{pmatrix} a & b \end{pmatrix} \times \begin{pmatrix} p & q \end{pmatrix} = \begin{pmatrix} ap+br & aq+bs \end{pmatrix}$$

Repeated Transformations:

$XT(P)$ means 'perform transformation T on P and then perform X on the image.'

$XX(P)$ may be written $X^2(P)$.

Inverse Transformations:

The inverse of a transformation is the transformation which takes the image back to the object.

If translation T has a vector $\begin{pmatrix} x \\ y \end{pmatrix}$, then the translation which has the opposite effect has vector $\begin{pmatrix} -x \\ -y \end{pmatrix}$.

This is written as T^{-1} .

If rotation R denotes 90° clockwise rotation about (0, 0), then R^{-1} denotes 90° anticlockwise rotation about (0, 0).

For all reflections, the inverse is the same reflection.

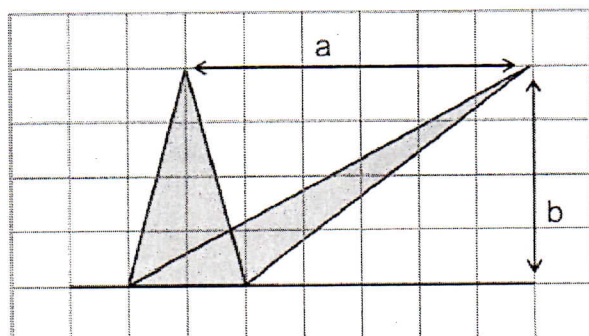
Base vectors

The base vectors are considered as $I = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$ and $J = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$

The columns of a matrix give us the images of I and J after the transformation.

Shear:

Shear factor = $\frac{\text{Distance a point moves due to the shear}}{\text{Perpendicular distance of the point from the fixed line}} = \frac{a}{b}$



[The shear factor will be the same calculated from any point on the object with the exception of those on the invariant line]

$$\text{Area of image} = \text{Area of object}$$

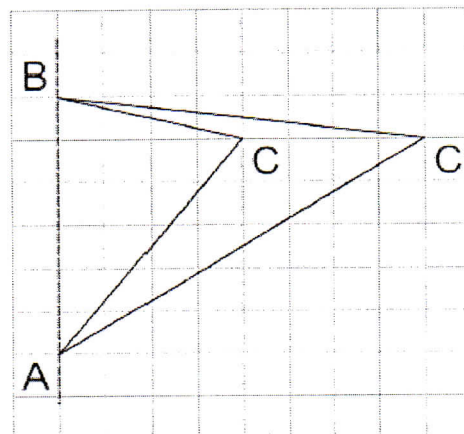
Stretch:

To describe a stretch, state;

- the stretch factor, p
- the invariant line,
- the direction of the stretch

(always perpendicular to the invariant line)

Scale factor = $\frac{\text{Perpendicular distance of } C' \text{ from } AB}{\text{Perpendicular distance of } C \text{ from } AB}$



Where, p is the stretch factor

$$\text{Area of image} = p \times \text{Area of object}$$

STATISTICS

Bar Graph:

A bar chart makes numerical information easy to see by showing it in a pictorial form.

The width of the bar has no significance. The length of each bar represents the quantity.

Pie Diagram:

The information is displayed using sectors of a circle.

Histograms:

A histogram displays the frequency of either continuous or grouped discrete data in the form of bars.

The bars are joined together.

The bars can be of varying width.

The frequency of the data is represented by the area of the bar and not the height.

[When class intervals are different it is the area of the bar which represents the frequency not the height]. Instead of frequency being plotted on the vertical axis, frequency density is plotted.

$$\text{Frequency density} = \frac{\text{frequency}}{\text{class width}}$$

Mean:

The mean of a series of numbers is obtained by adding the numbers and dividing the result by the number of numbers.

$$\text{Mean} = \frac{\sum fx}{\sum f} \quad \text{where } \sum fx \text{ means 'the sum of the products'}$$

i.e. $\sum (\text{number} \times \text{frequency})$

and $\sum f$ means 'the sum of the frequencies'.

Median:

The median of a series of numbers is obtained by arranging the numbers in ascending order and then choosing the number in the 'middle'. If there are two 'middle' numbers the median is the average (mean) of these two numbers.

Mode:

The mode of a series of numbers is simply the number which occurs most often.

Frequency tables:

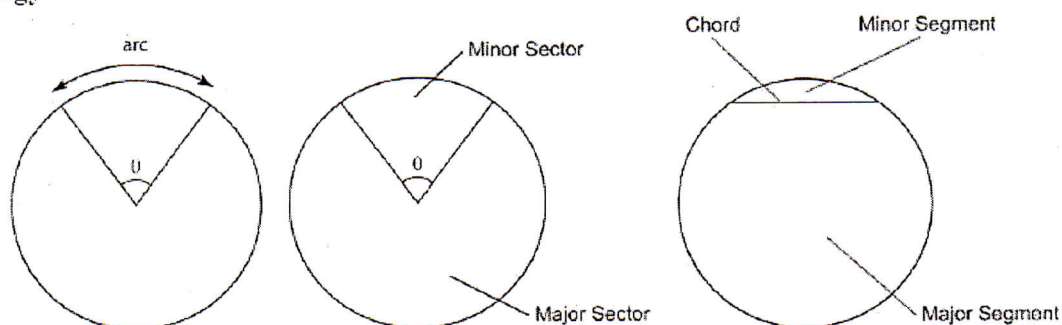
A frequency table shows a number x such as a score or a mark, against the frequency f or number of times that x occurs.

Symmetry:

- A line of symmetry divides a two-dimensional shape into two congruent (identical) shapes.
- A plane of symmetry divides a three-dimensional shape into two congruent solid shapes.
- A two-dimensional shape has rotational symmetry if, when rotated about a central point, it fits its outline. The number of times it fits its outline during a complete revolution is called the order of rotational symmetry.

| Shape | Number of Lines of Symmetry | Order of Rotational Symmetry |
|----------------------|-----------------------------|------------------------------|
| Square | 4 | 4 |
| Rectangle | 2 | 2 |
| Parallelogram | 0 | 2 |
| Rhombus | 2 | 2 |
| Trapezium | 0 | 1 |
| Kite | 1 | 1 |
| Equilateral Triangle | 3 | 3 |
| Regular Hexagon | 6 | 6 |

Terminology



Length and Area

| | | | | |
|--|--|---|--|--|
| <div><div><p>Length of Arc: $s = r \theta$</p><p>Length of Chord: $l = 2r \sin \frac{\theta}{2}$</p><p>Area of Sector: $A = \frac{1}{2} r^2 \theta$</p></div><div><p>Area of Segment: $A = \frac{1}{2} r^2 (\theta - \sin \theta)$</p><p>Area of Triangle: $A = \frac{1}{2} r^2 \sin \theta$</p></div></div> <div><p>r = radius A = area s = arc length θ = angle l = length of chord</p></div> | | | | |
| Arc Length: $s = r \theta$ | Length of chord: $l = 2r \sin \frac{\theta}{2}$ | Area of Sector: $A = \frac{1}{2} r^2 \theta$ | Area of Triangle: $A = \frac{1}{2} r^2 \sin \theta$ | Area of Segment: $A = \frac{1}{2} r^2 (\theta - \sin \theta)$ |