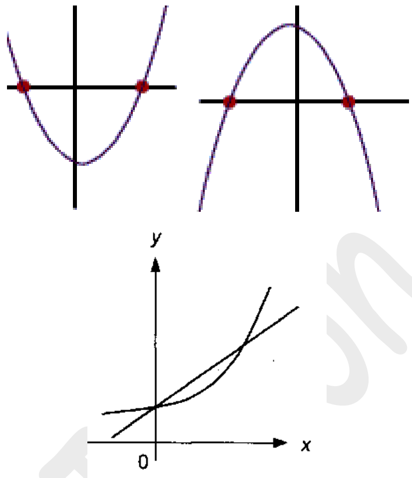
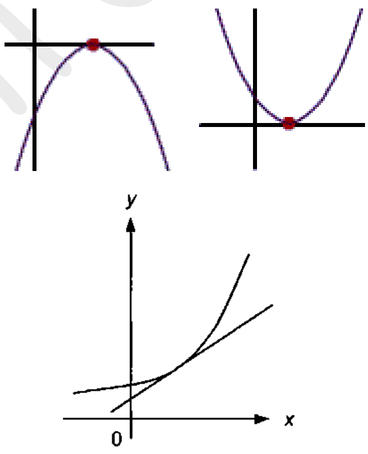
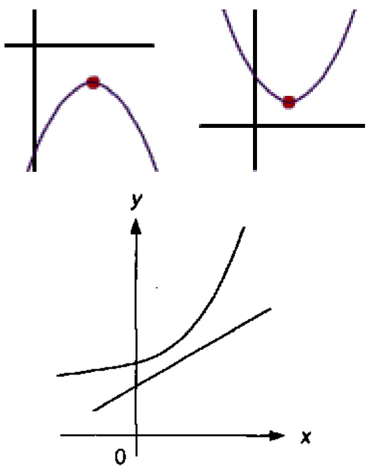
# N-Level Additional Math (4044) Formula List

*Formulas highlighted in yellow are found in the formula list of the exam paper.

| Quadratic Equation | |
|---|---|
| $b^2 - 4ac > 0$<br>Real and Distinct Roots<br>/ Unequal Roots | <br><br>Intersection at two different points |
| $b^2 - 4ac = 0$<br>Real and Equal Roots/Repeat Roots/ Coincident Roots. | <br><br>Tangency |
| $b^2 - 4ac < 0$<br>Imaginary roots<br>Also known as Complex Roots. | <br><br>Non-intersection |

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| **Roots of Quadratic Equation** | |
|---|---|
| The Quadratic Equation has solutions $\alpha$ and $\beta$ | |
| Sum of Roots is $\alpha + \beta = -\frac{b}{a}$ | Product of Roots is $\alpha\beta = \frac{c}{a}$ |
| Equation: $x^2$ – (Sum of Roots) x + (Product of Roots) = 0 | |
| $\alpha^2 + \beta^2 = (\alpha + \beta)^2 - 2\alpha\beta$ | $\alpha - \beta = \pm\sqrt{(\alpha - \beta)^2 - 4\alpha\beta}$ |
| $\alpha^4 + \beta^4 = (\alpha^2 + \beta^2)^2 - 2(\alpha\beta)^2$ | |

| **Indices** | | |
|---|---|---|
| **Same Base Number** | **Same Power** | |
| $x^a \times x^b = x^{a+b}$ | $a^m \times b^m = (a \times b)^m$ | Base number-Same → Power Add or Subtract |
| $\frac{x^a}{x^b} = x^{a-b}$ | $\frac{a^m}{b^m} = \left(\frac{a}{b}\right)^m$ | Power Same→ Base Number Multiply or Divide. |
| $\left(x^a\right)^b = x^{a \times b}$ **NOTE:** $\left(x^a\right)^b \neq x^a \times x^b$ | | |

| **Other Laws of Indices** | | |
|---|---|---|
| $x^{-a} = \frac{1}{x^a}$ | $x^{\frac{1}{b}} = \sqrt[b]{x^1}$ | $x^0 = 1$ |

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| | | |
|---|---|---|
| $x^a y^{-b} = \frac{x^a}{y^b}$ | $x^{-\frac{1}{b}} = \frac{1}{x^{\frac{1}{b}}} = \frac{1}{\sqrt[b]{x^1}}$ | |
| $\frac{1}{x^{-a}} = x^a$ | $\left(\frac{x}{y}\right)^{-a} = \left(\frac{y}{x}\right)^a$ | |
| $x^{\frac{a}{b}} = \sqrt[b]{x^a}$ | $x^{-\frac{a}{b}} = \frac{1}{x^{\frac{a}{b}}} = \frac{1}{\sqrt[b]{x^a}}$ | |

| **Surds** | |
|---|---|
| $\sqrt{a} \times \sqrt{b} = \sqrt{a \times b}$ | $\sqrt{a} \div \sqrt{b} = \sqrt{\frac{a}{b}}$ |
| $\sqrt{a} \times \sqrt{a} = \left(\sqrt{a}\right)^2 = a$ | $m\sqrt{a} \times n\sqrt{b} = m \times n\sqrt{a \times b}$ |
| $m\sqrt{a} + n\sqrt{a} = (m+n)\sqrt{a}$ | $m\sqrt{a} - n\sqrt{a} = (m-n)\sqrt{a}$ |
| **Rationalizing Denominator** | |
| $\frac{1}{n+\sqrt{a}} \times \frac{n-\sqrt{a}}{n-\sqrt{a}} = \frac{\left(n-\sqrt{a}\right)}{n^2-\left(\sqrt{a}\right)^2} = \frac{\left(n-\sqrt{a}\right)}{n^2-a}$ | |

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| |
|---|
| $\frac{1}{n-\sqrt{a}}\times\frac{n+\sqrt{a}}{n+\sqrt{a}}=\frac{\left(n+\sqrt{a}\right)}{n^2-\left(\sqrt{a}\right)^2}=\frac{\left(n-\sqrt{a}\right)}{n^2-a}$ |
| $\frac{1}{\sqrt{n}+\sqrt{a}}\times\frac{\sqrt{n}-\sqrt{a}}{\sqrt{n}-\sqrt{a}}=\frac{\left(n-\sqrt{a}\right)}{\left(\sqrt{n}\right)^2-\left(\sqrt{a}\right)^2}=\frac{\left(n-\sqrt{a}\right)}{n-a}$ |
| $\frac{1}{\sqrt{n}-\sqrt{a}}\times\frac{\sqrt{n}+\sqrt{a}}{\sqrt{n}+\sqrt{a}}=\frac{\left(n+\sqrt{a}\right)}{\left(\sqrt{n}\right)^2-\left(\sqrt{a}\right)^2}=\frac{\left(n+\sqrt{a}\right)}{n-a}$ |

## Polynomials & Partial Factions

$x^3+y^3=(x+y)\left(x^2-xy+y^2\right)$ $\quad$ $x^3-y^3=(x-y)\left(x^2+xy+y^2\right)$

## Binomial Theorem

$\binom{n}{r}=\frac{n!}{r!(n-r)!}$ $\qquad$ $\binom{n}{r}={}^{n}C_{r}$

**Remember:** $\binom{n}{0}=1,\ \binom{n}{1}=n,\ \binom{n}{2}=\frac{n(n-1)}{1\times 2},\ \binom{n}{3}=\frac{n(n-1)(n-2)}{1\times 2\times 3}$

**Since:**

$$(a+bx)^n=a^n+\binom{n}{1}a^{n-1}(bx)^1+\binom{n}{2}a^{n-2}(bx)^2+\binom{n}{3}a^{n-3}(bx)^3....\binom{n}{r}a^{n-r}(bx)^r+(bx)^n$$

**Therefore:**

$$(a+bx)^n = a^n + \frac{n}{1}a^{n-1}(bx)^1 + \frac{(n)(n-1)}{1\times 2}a^{n-2}(bx)^2 + \frac{(n)(n-1)(n-2)}{1\times 2\times 3}a^{n-3}(bx)^3 \ldots. + (bx)^n$$

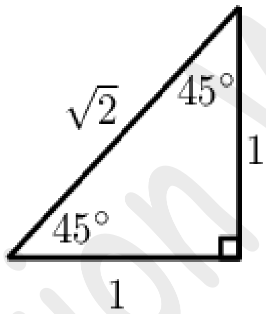
Alternate formula if a=1

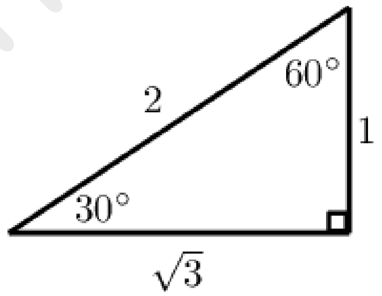
$$(1+kx)^n = \binom{n}{0}1^n(kx)^0 + \binom{n}{1}1^{n-1}(kx)^1 + \binom{n}{2}1^{n-2}(kx)^2 + \ldots.\binom{n}{n-1}1^{n-(n-1)}(kx)^{n-1} + \binom{n}{n}1^{n-n}(kx)^n$$

**OR**

$$(1+kx)^n = 1 + \binom{n}{1}(kx)^1 + \binom{n}{2}(kx)^2 + \ldots.\binom{n}{n-1}(kx)^{n-1} + (kx)^n$$

## Trigonometry




| | 30° | 45° | 60° |
|---|---|---|---|
| sin | $\frac{1}{2}$ | $\frac{1}{\sqrt{2}}$ | $\frac{\sqrt{3}}{2}$ |
| cos | $\frac{\sqrt{3}}{2}$ | $\frac{1}{\sqrt{2}}$ | $\frac{1}{2}$ |
| tan | $\frac{1}{\sqrt{3}}$ | 1 | $\sqrt{3}$ |

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**Principal Values of $\sin^{-1} x$, $\cos^{-1} x$, $\tan^{-1} x$,**

$-\frac{\pi}{2} \le \sin^{-1} x \le \frac{\pi}{2}$ $\quad 0 \le \cos^{-1} x \le \pi$ $\quad -\frac{\pi}{2} < \tan^{-1} x < \frac{\pi}{2}$

$\operatorname{cosec}\theta = \frac{1}{\sin\theta}$ $\quad \sec\theta = \frac{1}{\cos\theta}$ $\quad \cot\theta = \frac{1}{\tan\theta} = \frac{\cos\theta}{\sin\theta}$

$\sin^2\theta + \cos^2\theta = 1$ $\quad 1 + \cot^2\theta = \operatorname{cosec}^2\theta$ $\quad 1 + \tan^2\theta = \sec^2\theta$

Compound Angle Formula

$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$

$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$

$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$

Double Angle Formula

$\sin 2A = 2\sin A \cos A$

$\cos 2A = 2\cos^2 A - 1$ / $\cos 2A = \cos^2 A - \sin^2 A$ / $\cos 2A = 1 - 2\sin^2 A$

$\tan 2A = \frac{2\tan A}{1 - \tan^2 A}$

Half Angle Formula

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$$\sin\frac{A}{2} = \pm\sqrt{\frac{1-\cos A}{2}}$$

$$\cos\frac{A}{2} = \pm\sqrt{\frac{1+\cos A}{2}}$$

$$\tan\frac{A}{2} = \sqrt{\frac{1-\cos A}{1+\cos A}} = \frac{1-\cos A}{\sin A}$$

R-Formula

| | |
|---|---|
| $aCos\theta \pm bSin\theta = RCos(\theta \mp \alpha)$ | Where $R = \sqrt{a^2 + b^2}$ <br> $\tan\alpha = \frac{b}{a}$ |
| $aSin\theta \pm bCos\theta = RSin(\theta \pm \alpha)$ | |

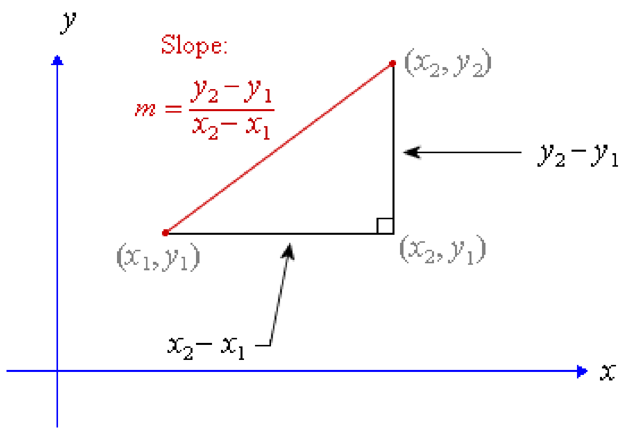
## Coordinate Geometry

Gradient(m) = $\frac{y_2 - y_1}{x_2 - x_1}$

General Equation $Y - y_1 = m(X - x_1)$

where $(x_1, y_1)$ is a point on the graph.

OR you may use y=mx+c



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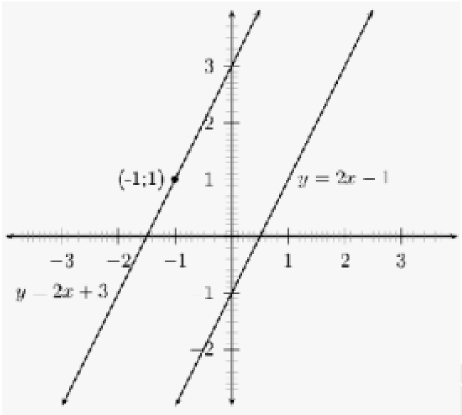
**Mid-point of a line**

$$\left(\frac{x_1+x_2}{2},\frac{y_1+y_2}{2}\right)$$

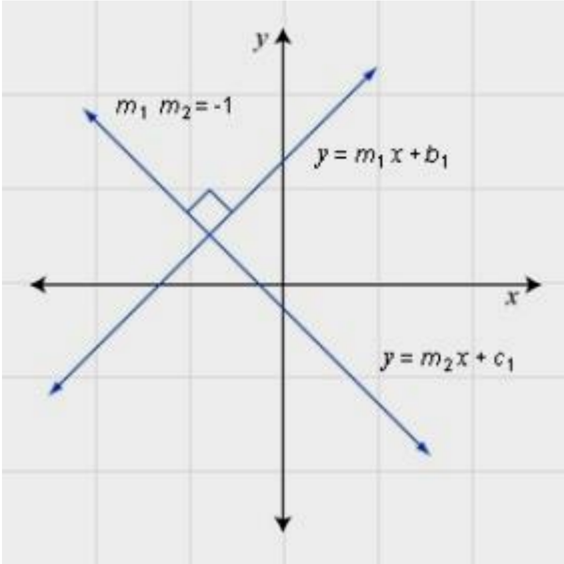
**Distance between two points**

$$\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$$

**When two lines have the same gradient**




$$m_1 = m_2$$



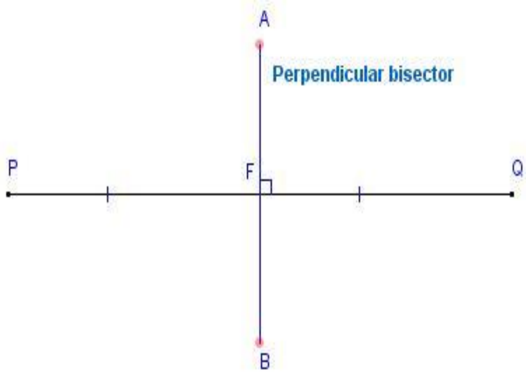

If two lines have perpendicular gradient i.e. 90° to each other.

$$m_1 = -\frac{1}{m_2} \quad \text{or} \quad m_1 \times m_2 = -1$$

**Perpendicular Bisector**

1. The two lines AB and PQ must intersect at 90°

$$m_1(AB) = -\frac{1}{m_2(PQ)}$$




2. One Line (AB) will cut the mid-point of the other line (PQ)

Mid-point PQ = $\left(\frac{x_2+x_1}{2},\frac{y_2+y_1}{2}\right)$

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Area of Plane Figure (Polygon Figure)

Vertices A($x_1$,$y_1$), B($x_2$,$y_2$),C ($x_3$,$y_3$)

$$\frac{1}{2}\begin{vmatrix} x_1 & x_2 & x_3 & x_4 & x_1 \\ y_1 & y_2 & y_3 & y_4 & y_1 \end{vmatrix}$$

$$\frac{1}{2}\left|(x_1y_2 + x_2y_3 + x_3y_4 + x_4y_1) - (x_2y_1 + x_3\,y_2 + x_4y_3 + x_1y_4)\right|$$

*It does NOT matter if you calculate in a clockwise or anti-clockwise direction.

The |modulus| in the formula will convert any negative values to positive.

## Circles

Radius of a circle

$$r = \sqrt{(x-a)^2 + (y-b)^2}$$

Equation of a Circle

$$(x-a)^2 + (y-b)^2 = r^2$$

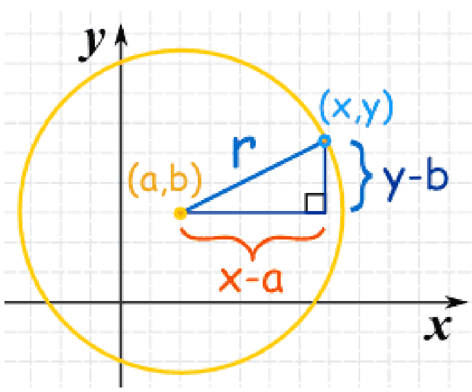
(Standard Form) or

$$x^2 + y^2 + 2gx + 2fy + c = 0$$

(General Form)

where $a = -g,\ b = -f$

and $r = \sqrt{g^2 + f^2 - c}$

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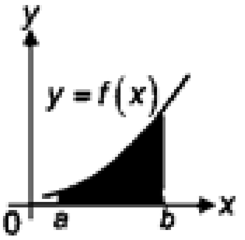
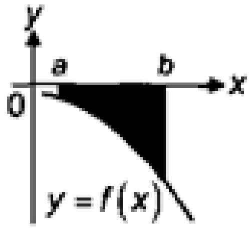
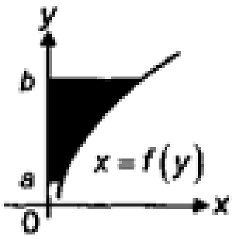
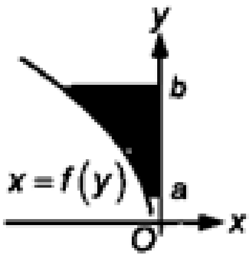
| **Differentiation** | | |
|---|---|---|
| $\frac{dy}{dx}(ax^n) = anx^{n-1}$ Where' a' and 'n' are constants. | $\frac{dy}{dx}(a) = 0$ | Sum/Difference of Function $\frac{d}{dx}(u \pm v) = \frac{du}{dx} \pm \frac{dv}{dx}$ |
| Chain Rule $\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx}$ | | Quotient Rule $\frac{d}{dx}\left(\frac{u}{v}\right) = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$ |
| Product Rule $\frac{d}{dx}(uv) = u\frac{dv}{dx} + v\frac{du}{dx}$ | | |

| **Integration** | | |
|---|---|---|
| $\int ax^n \, dx = \frac{ax^{n+1}}{n+1} + c$ where n≠-1 | $\int a \, dx = ax + c$ | $\int (ax+b)^n \, dx = \frac{(ax+b)^{n+1}}{(n+1)(a)} + c$ n≠1,a≠0; a & b are constants |
| Product of constant and a function $\int af(x)dx = a\int f(x)dx$ | | Sum and Difference of function $\int [\alpha f(x) \pm \beta g(x)]dx = \alpha\int f(x)dx \pm \beta\int g(x)dx$ |

| Integration of Area | |
|---|---|
| Area between Curve and X-axis $\int_a^b f(x)dx$   | Area between Curve and X-axis $-\int_a^b f(x)dx$   |
| Area between Curve and Y-axis $\int_a^b f(y)dy$   | Area between Curve and Y-axis $-\int_a^b f(y)dy$   |
| The End | |