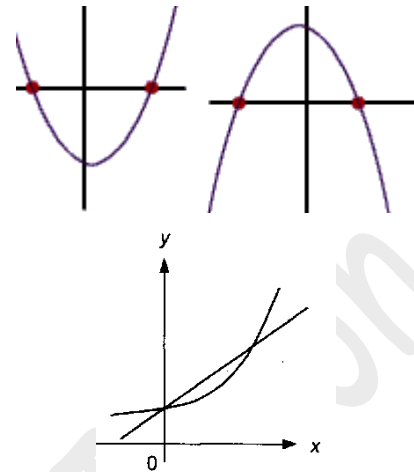
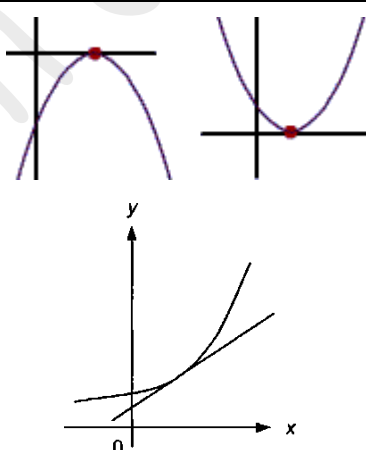
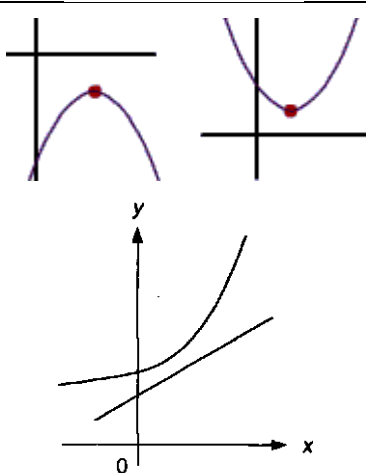


\*Formulas highlighted in yellow are found in the formula list of the exam paper.

Quadratic Equation	
<p><b><math>b^2 - 4ac &gt; 0</math></b> Real and Distinct Roots / Unequal Roots</p>	 <p>Intersection at two different points</p>
<p><b><math>b^2 - 4ac = 0</math></b> Real and Equal Roots/Repeat Roots/ Coincident Roots.</p>	 <p>Tangency</p>
<p><b><math>b^2 - 4ac &lt; 0</math></b> Imaginary roots Also known as Complex Roots.</p>	 <p>Non-intersection</p>

### Roots of Quadratic Equation

The Quadratic Equation has solutions  $\alpha$  and  $\beta$

Sum of Roots is  $\alpha + \beta = -\frac{b}{a}$

Product of Roots is  $\alpha\beta = \frac{c}{a}$

Equation:  $x^2 - (\text{Sum of Roots})x + (\text{Product of Roots}) = 0$

$$\alpha^2 + \beta^2 = (\alpha + \beta)^2 - 2\alpha\beta$$

$$\alpha - \beta = \pm\sqrt{(\alpha + \beta)^2 - 4\alpha\beta}$$

$$\alpha^4 + \beta^4 = (\alpha^2 + \beta^2)^2 - 2(\alpha\beta)^2$$

### Indices

Same Base Number

Same Power

$$x^a \times x^b = x^{a+b}$$

$$a^m \times b^m = (a \times b)^m$$

Base number-Same →  
Power Add or Subtract

$$\frac{x^a}{x^b} = x^{a-b}$$

$$\frac{a^m}{b^m} = \left(\frac{a}{b}\right)^m$$

Power Same →  
Base Number Multiply or Divide.

$$\left(x^a\right)^b = x^{a \times b}$$

**NOTE:**  $\left(x^a\right)^b \neq x^a \times x^b$

### Other Laws of Indices

$$x^{-a} = \frac{1}{x^a}$$

$$\frac{1}{x^b} = \sqrt[b]{x^1}$$

$$x^0 = 1$$

$x^a y^{-b} = \frac{x^a}{y^b}$	$x^{-\frac{1}{b}} = \frac{1}{x^{\frac{1}{b}}} = \frac{1}{\sqrt[b]{x^1}}$
$\frac{1}{x^{-a}} = x^a$	$\left(\frac{x}{y}\right)^{-a} = \left(\frac{y}{x}\right)^a$
$x^{\frac{a}{b}} = \sqrt[b]{x^a}$	$x^{-\frac{a}{b}} = \frac{1}{x^{\frac{a}{b}}} = \frac{1}{\sqrt[b]{x^a}}$

### Surds

$$\sqrt{a} \times \sqrt{b} = \sqrt{a \times b}$$

$$\sqrt{a} \div \sqrt{b} = \sqrt{\frac{a}{b}}$$

$$\sqrt{a} \times \sqrt{a} = (\sqrt{a})^2 = a$$

$$m\sqrt{a} \times n\sqrt{b} = m \times n \sqrt{a \times b}$$

$$m\sqrt{a} + n\sqrt{a} = (m+n)\sqrt{a}$$

$$m\sqrt{a} - n\sqrt{a} = (m-n)\sqrt{a}$$

### Rationalizing Denominator

$$\frac{1}{n + \sqrt{a}} \times \frac{n - \sqrt{a}}{n - \sqrt{a}} = \frac{(n - \sqrt{a})}{n^2 - (\sqrt{a})^2} = \frac{(n - \sqrt{a})}{n^2 - a}$$

$$\frac{1}{n-\sqrt{a}} \times \frac{n+\sqrt{a}}{n+\sqrt{a}} = \frac{(n+\sqrt{a})}{n^2 - (\sqrt{a})^2} = \frac{(n+\sqrt{a})}{n^2 - a}$$

$$\frac{1}{\sqrt{n}+\sqrt{a}} \times \frac{\sqrt{n}-\sqrt{a}}{\sqrt{n}-\sqrt{a}} = \frac{(n-\sqrt{a})}{(\sqrt{n})^2 - (\sqrt{a})^2} = \frac{(n-\sqrt{a})}{n-a}$$

$$\frac{1}{\sqrt{n}-\sqrt{a}} \times \frac{\sqrt{n}+\sqrt{a}}{\sqrt{n}+\sqrt{a}} = \frac{(n+\sqrt{a})}{(\sqrt{n})^2 - (\sqrt{a})^2} = \frac{(n+\sqrt{a})}{n-a}$$

### Polynomials & Partial Fractions

$$x^3 + y^3 = (x + y)(x^2 - xy + y^2) \quad x^3 - y^3 = (x - y)(x^2 + xy + y^2)$$

### Linear Factor

$$\frac{mx+n}{(ax+b)(cx-d)} = \frac{A}{(ax+b)} + \frac{B}{(cx+d)}$$

### Repeat Factors

$$\frac{mx+n}{(ax+b)(cx-d)^2} = \frac{A}{(ax+b)} + \frac{B}{(cx+d)} + \frac{C}{(cx+d)^2}$$

Quadratic Factors

$$\frac{mx+n}{(ax+b)(cx^2-d)} = \frac{A}{(ax+b)} + \frac{Bx+C}{(cx^2+d)}$$

**Note:** If the highest coefficient of the NUMERATOR is the SAME or LARGER than the DENOMINATOR. Do **LONG DIVISION** before partial fractions.

**Logarithm**

$\log_b 1 = 0$	$\ln 1 = 0$	$b^y = x$ $y = \log_b x$  <b>Remember:</b> 1) $\ln x = \log_e x$  2) $\log_e e = 1$  $\ln e = 1$
$\log_b b = 1$	$\ln_e x = x$	
$\log_b b^x = x$	$e^{\ln a} = a$	
$\log_b m + \log_b n = \log_b (m \times n)$  <b>Note:</b> $\log_b m + \log_b n \neq \log_b m \times \log_b n$		
$\log_b m - \log_b n = \log_b \left(\frac{m}{n}\right)$		
$\log_b m^a = a \times \log_b m$		

$$\log_v u = \frac{\log_a u}{\log_a v}$$

$$\log_v u = \frac{\log_u u}{\log_u v} = \frac{1}{\log_u v}$$

### Binomial Theorem

$$\binom{n}{r} = \frac{n!}{r!(n-r)!} \quad \binom{n}{r} = {}^n C_r$$

**Remember:**  $\binom{n}{0} = 1$ ,  $\binom{n}{1} = n$ ,  $\binom{n}{2} = \frac{n(n-1)}{1 \times 2}$ ,  $\binom{n}{3} = \frac{n(n-1)(n-2)}{1 \times 2 \times 3}$

**Since:**

$$(a+bx)^n = a^n + \binom{n}{1} a^{n-1} (bx)^1 + \binom{n}{2} a^{n-2} (bx)^2 + \binom{n}{3} a^{n-3} (bx)^3 \dots \binom{n}{r} a^{n-r} (bx)^r + (bx)^n$$

**Therefore:**

$$(a+bx)^n = a^n + \frac{n}{1} a^{n-1} (bx)^1 + \frac{(n)(n-1)}{1 \times 2} a^{n-2} (bx)^2 + \frac{(n)(n-1)(n-2)}{1 \times 2 \times 3} a^{n-3} (bx)^3 \dots + (bx)^n$$

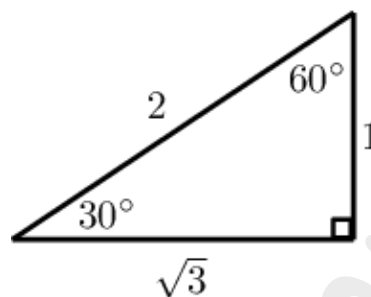
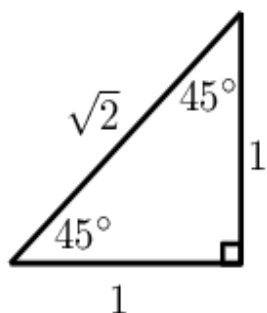
**Alternate formula if a=1**

$$(1+kx)^n = \binom{n}{0} 1^n (kx)^0 + \binom{n}{1} 1^{n-1} (kx)^1 + \binom{n}{2} 1^{n-2} (kx)^2 + \dots \binom{n}{n-1} 1^{n-(n-1)} (kx)^{n-1} + \binom{n}{n} 1^{n-n} (kx)^n$$

**OR**

$$(1+kx)^n = 1 + \binom{n}{1} (kx)^1 + \binom{n}{2} (kx)^2 + \dots \binom{n}{n-1} (kx)^{n-1} + (kx)^n$$

**Trigonometry**



	$30^\circ$	$45^\circ$	$60^\circ$
sin	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{\sqrt{3}}{2}$
cos	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$
tan	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$

**Principal Values of  $\sin^{-1} x$ ,  $\cos^{-1} x$ ,  $\tan^{-1} x$ ,**

$$-\frac{\pi}{2} \leq \sin^{-1} x \leq \frac{\pi}{2} \quad 0 \leq \cos^{-1} x \leq \pi \quad -\frac{\pi}{2} < \tan^{-1} x < \frac{\pi}{2}$$

$$\operatorname{cosec} \theta = \frac{1}{\sin \theta} \quad \sec \theta = \frac{1}{\cos \theta} \quad \cot \theta = \frac{1}{\tan \theta} = \frac{\cos \theta}{\sin \theta}$$

$$\sin^2 \theta + \cos^2 \theta = 1 \quad 1 + \cot^2 \theta = \operatorname{cosec}^2 \theta \quad 1 + \tan^2 \theta = \sec^2 \theta$$

**Compound Angle Formula**

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$

### Double Angle Formula

$$\sin 2A = 2 \sin A \cos A$$

$$\cos 2A = 2 \cos^2 A - 1 \quad / \quad \cos 2A = \cos^2 A - \sin^2 A \quad / \quad \cos 2A = 1 - 2 \sin^2 A$$

$$\tan 2A = \frac{2 \tan A}{1 - \tan^2 A}$$

### Half Angle Formula

$$\sin \frac{A}{2} = \pm \sqrt{\frac{1 - \cos A}{2}}$$

$$\cos \frac{A}{2} = \pm \sqrt{\frac{1 + \cos A}{2}}$$

$$\tan \frac{A}{2} = \sqrt{\frac{1 - \cos A}{1 + \cos A}} = \frac{1 - \cos A}{\sin A}$$

### R-Formula

$$a \cos \theta \pm b \sin \theta = R \cos(\theta \mp \alpha)$$

$$a \sin \theta \pm b \cos \theta = R \sin(\theta \pm \alpha)$$

Where  $R = \sqrt{a^2 + b^2}$

$$\tan \alpha = \frac{b}{a}$$



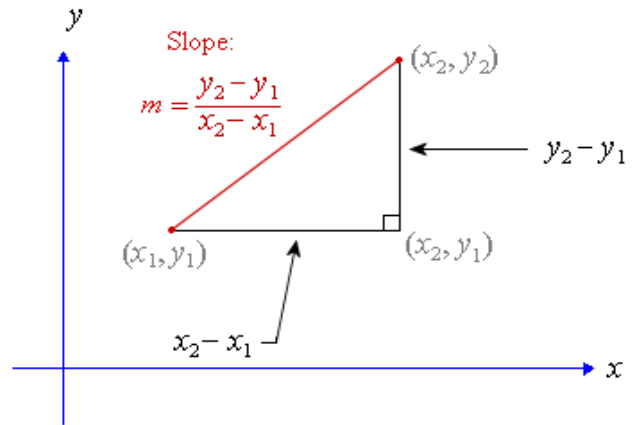
### Coordinate Geometry

$$\text{Gradient}(m) = \frac{y_2 - y_1}{x_2 - x_1}$$

$$\text{General Equation } Y - y_1 = m(X - x_1)$$

where  $(x_1, y_1)$  is a point on the graph.

OR you may use  $y = mx + c$



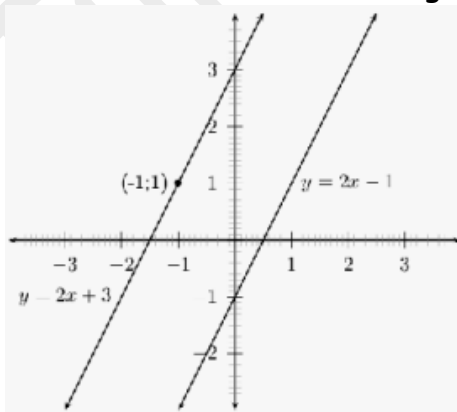
Mid-point of a line

$$\left( \frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right)$$

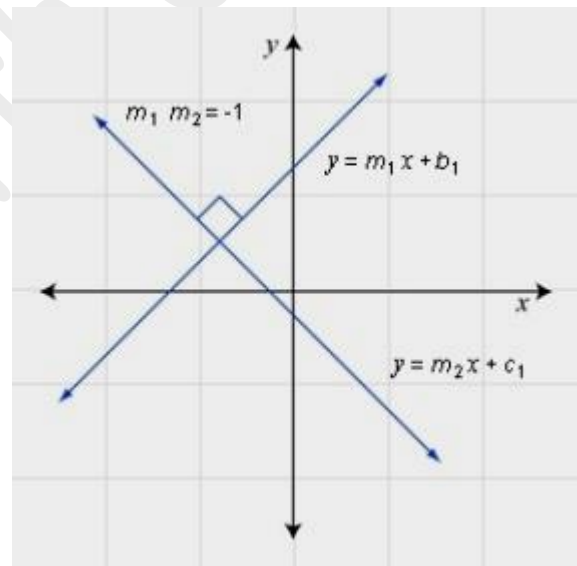
Distance between two points

$$\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

When two lines have the same gradient



$$m_1 = m_2$$



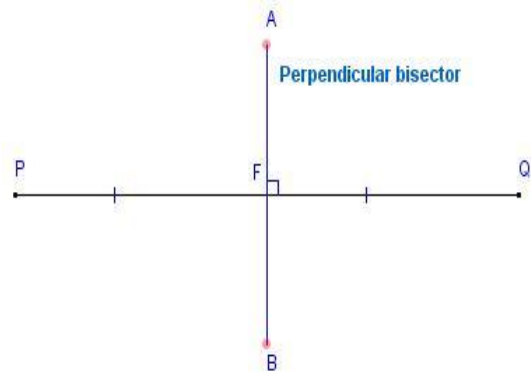
If two lines have perpendicular gradient i.e.  $90^\circ$  to each other.

$$m_1 = -\frac{1}{m_2} \quad \text{or} \quad m_1 \times m_2 = -1$$

Perpendicular Bisector

1. The two lines  $AB$  and  $PQ$  must intersect at  $90^\circ$

$$m_1(AB) = -\frac{1}{m_2(PQ)}$$



2. One Line ( $AB$ ) will cut the mid-point of the other line ( $PQ$ )

$$\text{Mid-point } PQ = \left( \frac{x_2 + x_1}{2}, \frac{y_2 + y_1}{2} \right)$$

Area of Plane Figure (Polygon Figure)

Vertices  $A(x_1, y_1), B(x_2, y_2), C(x_3, y_3)$

$$\frac{1}{2} \begin{vmatrix} x_1 & x_2 & x_3 & x_4 & x_1 \\ y_1 & y_2 & y_3 & y_4 & y_1 \end{vmatrix}$$

$$\frac{1}{2} |(x_1y_2 + x_2y_3 + x_3y_4 + x_4y_1) - (x_2y_1 + x_3y_2 + x_4y_3 + x_1y_4)|$$

\*It does NOT matter if you calculate in a clockwise or anti-clockwise direction.

The |modulus| in the formula will convert any negative values to positive.

## Circles

Radius of a circle

$$r = \sqrt{(x-a)^2 + (y-b)^2}$$

Equation of a Circle

$$(x - a)^2 + (y - b)^2 = r^2$$

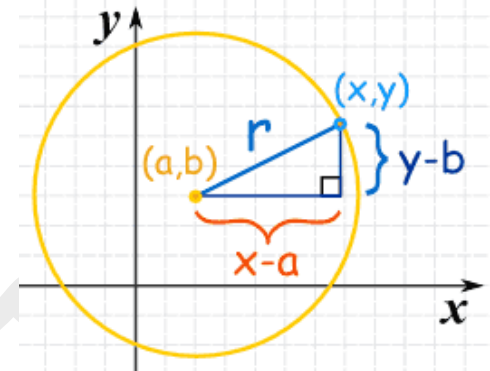
(Standard Form) or

$$x^2 + y^2 + 2gx + 2fy + c = 0$$

(General Form)

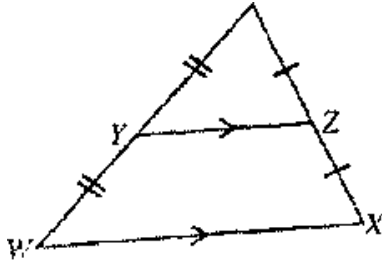
where  $a = -g$ ,  $b = -f$

and  $r = \sqrt{g^2 + f^2 - c}$



Proofs in Plane Geometry

Midpoint Theorem

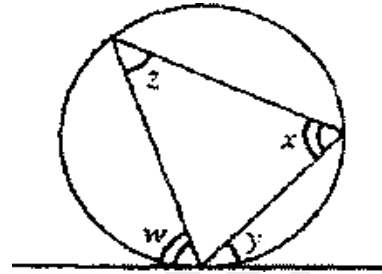


If X and Y are midpoints,  
then  $YZ \parallel WX$

$$YZ = \frac{1}{2} WX$$

Tangent -Chord Theorem

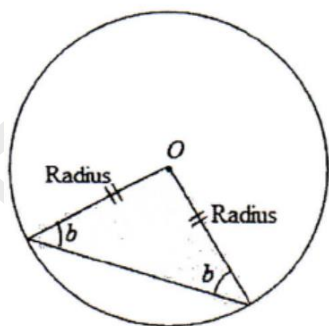
(Alternate Segment Theorem)



Angle W = Angle X

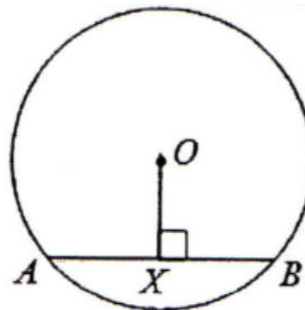
Angle Y = Angle Z

Isosceles Triangle

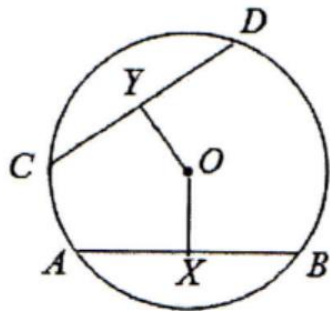


Perpendicular from Centre Bisects  
Chord

$$\angle OXA = \angle OXB = 90^\circ$$

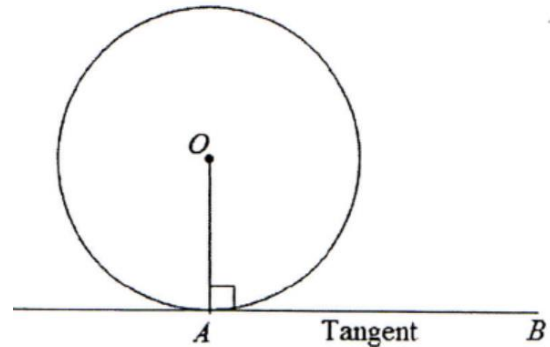


Equal Chord, Equal Distance from Centre



Tangent Perpendicular Radius

$$\angle OAB = 90^\circ$$

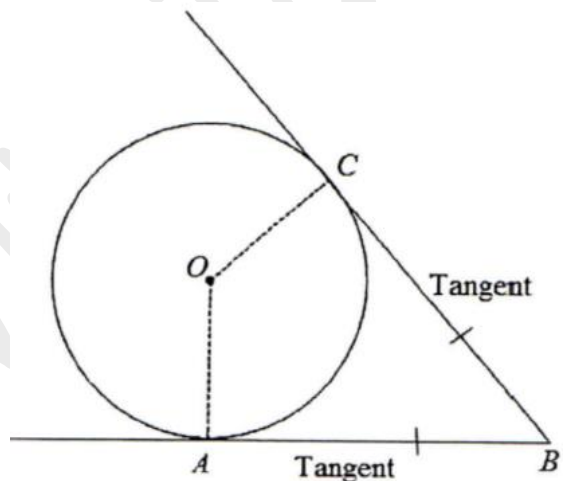


Tangents from External Point

$$BC = BA$$

$$\angle OCB = \angle OAB = 90^\circ$$

$$OA = OC \text{ (radius)}$$



### Differentiation

$$\frac{dy}{dx}(ax^n) = anx^{n-1}$$

Where 'a' and 'n' are constants.

$$\frac{dy}{dx}(a) = 0$$

Sum/Difference of Function

$$\frac{d}{dx}(u \pm v) = \frac{du}{dx} \pm \frac{dv}{dx}$$

Chain Rule

$$\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx}$$

Quotient Rule

$$\frac{d}{dx} \left( \frac{u}{v} \right) = \frac{v \frac{du}{dx} - u \frac{dv}{dx}}{v^2}$$

Product Rule

$$\frac{d}{dx} (uv) = u \frac{dv}{dx} + v \frac{du}{dx}$$

$$\frac{d}{dx} (\sin x) = \cos x$$

$$\frac{d}{dx} (\tan x) = \sec^2 x$$

$$\frac{d}{dx} (\cos x) = -\sin x$$

Use Chain Rule to differentiate the functions below.

$$\frac{d}{dx} [a \sin (bx + c)] = a \times \cos (bx + c) \times b$$

$$\frac{d}{dx} [a \cos (bx + c)] = a \times -\sin (bx + c) \times b$$

$$\frac{d}{dx} [a \tan (bx + c)] = a \times \sec^2 (bx + c) \times b$$

$$\frac{d}{dx} [a \sin^n (bx + c)] = a \times n \times \sin^{n-1} (bx + c) \times \cos (bx + c) \times b$$

$$\frac{d}{dx} [a \cos^n (bx + c)] = a \times n \times \cos^{n-1} (bx + c) \times -\sin (bx + c) \times b$$

$$\frac{d}{dx} [a \tan^n (bx + c)] = a \times n \times \tan^{n-1} (bx + c) \times \sec^2 (bx + c) \times b$$

Exponential/Natural Logarithm Function

$$\frac{d}{dx} (e^{ax+b}) = ae^{ax+b}$$

'a' is a constant

$$\frac{d}{dx} (e^x) = e^x$$

$$\frac{d}{dx} (\ln x) = \frac{1}{x} \quad (\text{where } x > 0)$$

$$\frac{d}{dx} [\ln(ax + b)] = \frac{a}{ax + b}$$

(where  $ax + b > 0$ )

Integration

$$\int ax^n dx = \frac{ax^{n+1}}{n+1} + c$$

where  $n \neq -1$

$$\int a dx = ax + c \quad \int (ax + b)^n dx = \frac{(ax + b)^{n+1}}{(n+1)(a)} + c$$

$n \neq -1, a \neq 0$ ; a & b are constants

Product of constant and a function

$$\int af(x) dx = a \int f(x) dx$$

Sum and Difference of function

$$\int [\alpha f(x) \pm \beta g(x)] dx = \alpha \int f(x) dx \pm \beta \int g(x) dx$$

$\int a \cos bx \, dx = \frac{a \sin bx}{b} + c$	$\int a \sin bx \, dx = -\frac{a \cos bx}{b} + c$	$\int a \sec^2 bx \, dx = \frac{a \tan bx}{b} + c$
$\int \frac{1}{x} \, dx = \ln x + c$ where $x > 0$		$\int \frac{1}{ax + b} \, dx = \frac{\ln(ax + b)}{a} + C$
$\int \frac{1}{ax^n + b} \, dx = \frac{\ln(ax^n + b)}{a \times n \times x^{n-1}} + C$		
$\int e^x \, dx = e^x + c$	$\int e^{ax+b} \, dx = \frac{e^{ax+b}}{a} + C$	
$\int e^{ax^n+b} \, dx = \frac{e^{ax^n+b}}{a \times n \times x^{n-1}} + C$		

### Integration of Area

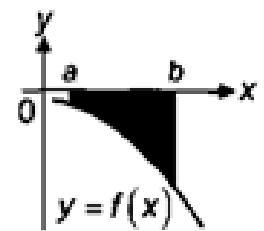
Area between  
Curve and X-axis

$$\int_a^b f(x) \, dx$$



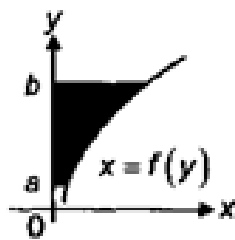
Area between  
Curve and X-axis

$$-\int_a^b f(x) \, dx$$



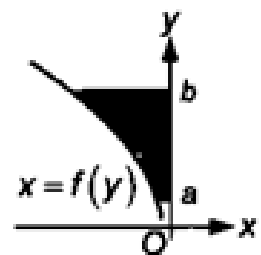
Area between  
Curve and Y-axis

$$\int_a^b f(y) \, dy$$



Area between  
Curve and Y-axis

$$-\int_a^b f(y) \, dy$$





**Kinematics**

Velocity is the RATE of CHANGE of Displacement

$$v = \frac{ds}{dt} \quad \text{or}$$

$$v = \int a \, dt = \int \frac{dv}{dt} \, dt$$

where v:velocity, s:displacement,  
t:time

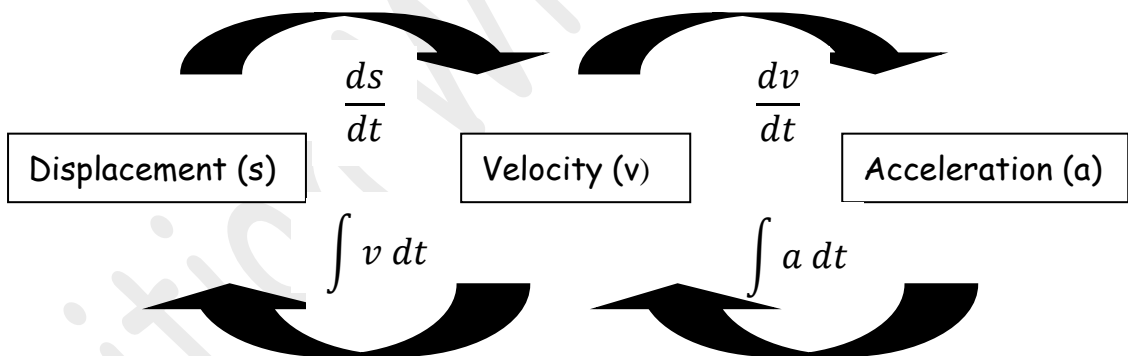
Acceleration is the RATE of CHANGE of Velocity

$$a = \frac{dv}{dt}$$

where a:acceleration

Some other methods to find acceleration

$$a = \frac{d^2s}{dt^2} \quad a = \frac{dv}{ds} \times \frac{ds}{dt}$$



The End